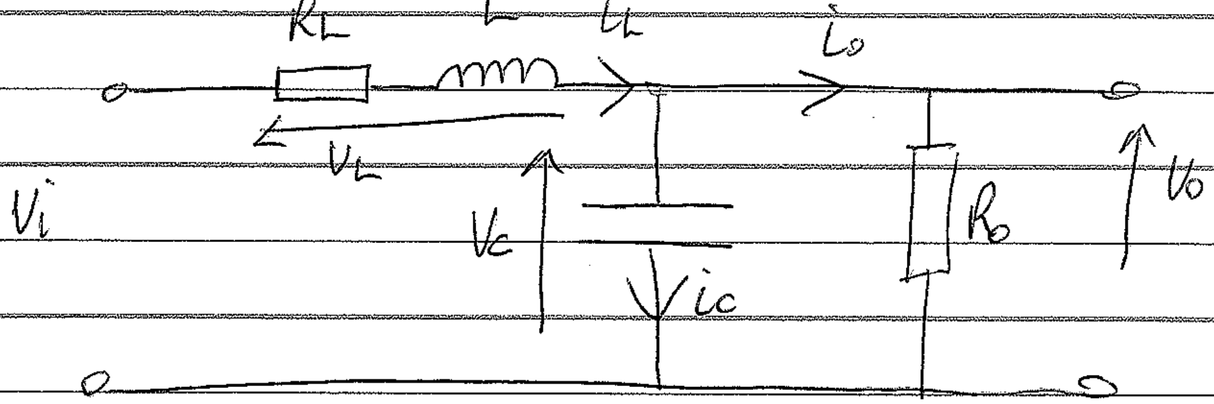
things to do today

date

1



2

3

4 $V_i - V_L - V_C = 0 \qquad V_C = V_O$

$V_i - V_L = V_C$

5

$$V_C = \frac{1}{C}\int i_C \, dt$$

6

7 $i_L = i_C + i_o$

$i_C = i_L - i_o$

8

$$V_C = \frac{1}{C}\int i_L - i_o \, dt$$

9

10 $i_o = \frac{V_C}{R_o}$

11

$$V_C = \frac{1}{C}\int i_L - \frac{1}{CR_o}\int V_C$$

12

Banner

product code 909-5002

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1 $$v_C = \frac{1}{C}\int i_L - \int \frac{v_C}{CR_o} \, dt$$

2

3 $$v_C = \frac{1}{C}\int i_L \, dt - \frac{1}{CR_o}\int v_C \, dt$$

4

5 $$v_C + \frac{1}{CR_o}\int v_C \, dt = \frac{1}{C}\int i_L \, dt$$

6

7 $$\frac{dv_C}{dt} + \frac{v_C}{CR_o} = \frac{i_L}{C} \quad ; \quad v_L = L\frac{di_L}{dt}$$

$$\frac{1}{L}\int v_L = i_L$$

8

9 $$\therefore \frac{dv_C}{dt} + \frac{v_C}{CR_o} = \frac{1}{LC}\int v_L \, dt$$

$$v_i - v_L = v_C$$

10 $$v_i - v_C = v_L$$

11 $$\frac{dv_C}{dt} + \frac{v_C}{CR_o} = \frac{1}{LC}\int (v_i - v_C) \, dt$$

12 $$\frac{dv_C}{dt} + \frac{v_C}{CR_o} - \frac{1}{LC}\int v_i \, dt - \frac{1}{LC}\int v_C \, dt$$

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1 $$LC \frac{dV_c}{dt} + \frac{L\cancel{C} V_c}{\cancel{C} R_o} = \int V_i \, dt - \int V_c \, dt$$

2

Differentiating again

3

$$LC \frac{d^2V_c}{dt^2} + \frac{L\cancel{C}}{R_o} \frac{dV_c}{dt} = V_i - V_c$$

4

5 $$V_c + \frac{L\cancel{C}}{R_o} \frac{dV_c}{dt} + LC \frac{d^2V_c}{dt^2} = V_i$$

6

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$$LC\frac{d^2V_c}{dt^2} + \frac{\cancel{L}}{R_0}\frac{dV_c}{dt} + V_c = V_i(t)$$

$$V_c(t) = V_{c\,cf}(t) + V_{c\,pi}(t)$$

$V_{c\,cf}(t)$ can be found by putting $V_i(t) = 0$

$$\therefore \frac{LC\,d^2V_c(t)}{dt^2} + \frac{\cancel{L}}{R_0}\frac{dV_c(t)}{dt} + V_c(t) = 0$$

It is a well known fact that the solution to this second order differential equation is of the form

$$V_c(t) = Ae^{s_1t} + Be^{s_2t}$$

Where A+B are two arbitrary constants and s_1 and s_2 are the roots of a quadratic equation. This quadratic equation is called the auxiliary equation and is obtained from

$$LC\frac{d^2V_c(t)}{dt^2} + \frac{\cancel{L}}{R_0}\frac{dV_c(t)}{dt} + V_c(t) = 0$$

by writing $s^2 = \frac{d^2V_c(t)}{dt^2}$; $s = \frac{dV_c(t)}{dt}$

$1 = V_c(t)$.

$V_{c\,cf}(t)$ ~~V_c~~ $= e^{\alpha t}\left\{ K_1\cos\beta t + K_2\sin\beta t \right\}$

We now wish to find the P.I. so that we can get ~~a~~ the overall equation for $V_c(t)$

$$\text{i.e. } V_c(t) = V_{c\,cf}(t) + V_{c\,pi}(t)$$

$V_{c\,pi}(t)$

We do this by solving the equation

$$LC\frac{d^2V_c(t)}{dt^2} + \frac{L}{R_0}\frac{dV_c(t)}{dt} + V_c(t) = V_i(t)$$

and assume that the forcing function $V_i(t)$ is of a particular form. In this case assume $V_i(t)$ is ~~a~~ of a ~~tr~~ trigonometric form i.e.

$$V_i(t) = \cancel{K\sin t}\ K\sin\omega t$$

Assuming this is the case then

$$V_{c\,pi}(t) = C\cos t + D\sin t$$

$$\frac{dV_{c\,pi}(t)}{dt} = -C\sin t + D\cos t$$

$$\frac{d^2V_{c\,pi}(t)}{dt^2} = -C\cos t - D\sin t$$

$$V_{c\,P_I}(t) = C\cos t + D\sin t$$

$R = 10$

$$V_{c\,PI}(t) = 1\times10^{-5}\cos t + \sin t \quad \left\{ = \frac{1\times10^{-4}}{R}\cos t + \sin t \right\}$$

$R = 100$

$$V_{c\,PI}(t) = 1\times10^{-6}\cos t + \sin t$$

$R = 1000$

$$V_{c\,PI}(t) = 1\times10^{-7}\cos t + \sin t.$$

It can be seen that even @ low R values the term in cost is insignificant with a maximum value of:

$R=10$ $\quad 1\times10^{-5}\cos t \quad$ max. value $= 1\times10^{-5}$
$= 0.00001$

$R = 10$

$$V_c(t) = e^{\alpha t}\left\{ K_1 \cos\beta t + K_2 \sin\beta t \right\} + \frac{1\times10^{-4}}{R}\cos t + \sin t$$

We assumed the forcing function was of the form:
~~$V_i(t)$~~

$$V_i(t) = k\sin t$$

and then assumed $k = 1$

If we now assume $k = 23.5$ what is the difference?

$$D - \frac{L}{R} \cdot C - LC \cdot D = 23.5$$

From the quadratic spreadsheet we get

$$D = 23.5$$

$R = 10$ $\quad$ $C = 2.35 \times 10^{-5}$
$R = 100$ $\quad$ $C = 2.35 \times 10^{-6}$
$R = 1000$ $\quad$ $C = 2.35 \times 10^{-7}$

$$V_{cp}(t) = \frac{2.35 \times 10^{-4}}{R} \cos t + 23.5 \sin t$$

Finding Constants of Complementary Function

$$V_{c\,cf}(t) = e^{at}\left\{K_1 \cos\beta t + K_2 \sin\beta t\right\}$$

a + b are roots of the ~~auth~~ auxiliary equation which depend on the load resistance R.

Assume $V_i(t) = k \sin \omega t$

$k = 23.5$

$\omega = 2\pi f$; $f = 43000$ Hz

$$\therefore V_i(t) = 23.5 \sin 43 \times 10^3 t$$

$$\therefore V_c(t) = e^{at}\left\{K_1 \cos\beta t + K_2 \sin\beta t\right\} + \frac{2.35 \times 10^{-4}}{R} \cos 2\pi \times 43000 t + 23.5 \sin 2\pi \times 43000 t$$

i.e. two unknowns (constants) need to be found K₁ and K₂. Therefore we need to know two values for $V_c(t)$ at particular times. One perhaps might be

$V_c(0^+) = 0$ another one might be

$$\frac{dV_c(0^+)}{dt} = ?$$

$$S_1 = e^{-227t}(\cos 21{,}319t + \sin 21{,}319t)$$
$$S_2 = e^{-227t}(\cos 21{,}319t - \sin 21{,}319t)$$

$$y_{CF}(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}$$
$$y_{CF}(t) = e^{\alpha t}\left\{ A\cos\beta t + j B\sin\beta t \right\}$$

$A = K_1 + K_2$
$B = K_1 - K_2$
$\alpha = -227$
$\beta = 21{,}319$

The constants K_1 and K_2 are obtained from the initial conditions

$$i_L(0^-) = 0 \quad V_C(0^-) = 0$$

We want $y(0^+)$ and $\frac{dy}{dt}(0^+)$ i.e. conditions @ $t = 0^+$

Since capacitor voltage does not change instantaneously

$$V_C(0^+) = y(0^+) = 0$$