

$f_c = 2000$

$$X(k) = e^{-j\pi k(K-1)/N} \frac{\sin(\pi k K/N)}{\sin(\pi k/N)}$$

LEAKAGE

$f_c =$

$F_s =$

$N =$

$f_k =$

$T =$

$f_a =$ actual frequency

		Sampling Frequency, F_s		$I = \text{INTEGER}$		$N \times f_c$	
$f_k =$	$k =$	N	K	$= 256000$		F_s	
				Re	Im	MODULUS	
0	0	128	64				
2000	1	128	64	0.02454123	0.999699	1.0000	1.0000 0.0245
4000	2						
6000	3	128	64	-0.0735646	-0.99729	1.0000	-1.0000 0.0736
8000	4						
10000	5	128	64	0.12241068	0.99248	1.0000	1.0000 0.1224
12000	6						
14000	7	128	64	-0.1709619	-0.98528	1.0000	-1.0000 0.1710
16000	8						
18000	9	128	64	0.21910124	0.975702	1.0000	1.0000 0.2191
20000	10						
22000	11	128	64	-0.2667128	-0.96378	1.0000	-1.0000 0.2667
24000	12						
26000	13	128	64	0.31368174	0.949528	1.0000	1.0000 0.3137
28000	14						
30000	15	128	64	-0.359895	-0.93299	1.0000	-1.0000 0.3599
32000	16	128					
34000	17	128	64	0.40524131	0.91421	1.0000	1.0000 0.4052
36000	18						
38000	19	128	64	-0.4496113	-0.89322	1.0000	-1.0000 0.4496
40000	20						
42000	21	128	64	0.49289819	0.870087	1.0000	1.0000 0.4929
44000	22						
46000	23	128	64	-0.5349976	-0.84485	1.0000	-1.0000 0.5350
48000	24						
50000	25	128	64	0.57580819	0.817585	1.0000	1.0000 0.5758
52000	26						
54000	27	128	64	-0.6152316	-0.78835	1.0000	-1.0000 0.6152
56000	28						
58000	29	128	64	0.65317284	0.757209	1.0000	1.0000 0.6532
60000	30						
62000	31	128	64	-0.6895405	-0.72425	1.0000	-1.0000 0.6895
64000	32	128	64	0.70710678	-0.70711	1.0000	0.0000 0.7071
			0				

2000

256000

128

$k \cdot F_s / N$ $k = 0, 1, 2 \dots$

0.0005

1

			<u>FOURIER</u>	2 line	1 line	<u>FOURIER</u>	<u>FOURIER</u>	
			<u>TRANSFORM</u>	spectra	spectra	1 line	2 line	MATLAB
			$X(f) \cdot T/N$	$X(f)/N$	$X(f)/N$	spectra	spectra	"0 TO +128
$X(f)$	$X(f)/N$	$X(f) \cdot T/N$	dBm	dBm	dBm	dBm	dBm	dBm
64.000	0.500	2.500E-04	-42.041	23.979	23.979	23.979	23.9794	23.8430
40.7478	0.3183	1.592E-04	-45.963	20.058	23.068	23.067	20.0570	25.0460
0.0000	0.0000			0.000		0.000	0.0000	19.5320
13.5935	0.1062	5.310E-05	-55.498	10.522	13.533	13.525	10.5146	13.5250
0.0000	0.0000			0.000		0.000	0.0000	11.5880
8.1692	0.0638	3.191E-05	-59.921	6.099	9.110	9.088	6.0776	9.1020
0.0000	0.0000			0.000		0.000	0.0000	7.7680
5.8493	0.0457	2.285E-05	-62.823	3.198	6.208	6.165	3.1550	6.2000
0.0000	0.0000			0.000		0.000	0.0000	5.1860
4.5641	0.0357	1.783E-05	-64.978	1.043	4.053	3.982	0.9722	4.0450
	0.0000			0.000		0.000	0.0000	3.2300
3.7494	0.0293	1.465E-05	-66.686	-0.665	2.345	2.239	-0.7709	2.3370
	0.0000			0.000		0.000	0.0000	1.6590
3.1879	0.0249	1.245E-05	-68.095	-2.074	0.936	0.788	-2.2219	0.9280
	0.0000			0.000				0.349
2.7786	0.0217	1.085E-05	-69.288	-3.268	-0.257			-0.265
0.000	0.000			0.000				-0.768
2.4677	0.0193	9.639E-06	-70.319	-4.298	-1.288			-1.296
0.0000	0.0000			0.000				0.000
2.2241	0.0174	8.688E-06	-71.222	-5.201				0.000
0.0000	0.0000			0.000				0.000
2.0288	0.0159	7.925E-06	-72.020	-5.999				0.000
0.0000	0.0000			0.000				0.000
1.8692	0.0146	7.301E-06	-72.732	-6.711				0.000
0.0000	0.0000			0.000				0.000
1.7367	0.0136	6.784E-06	-73.370	-7.350				0.000
	0.0000			0.000				0.000
1.6254	0.0127	6.349E-06	-73.946	-7.925				0.000
	0.0000			0.000				0.000
1.5310	0.0120	5.980E-06	-74.465	-8.445				0.000
	0.0000			0.000				0.000
1.4502	0.0113	5.665E-06	-74.936	-8.915				0.000
0.0000	0.0000	1.083E-20	-369.308	-303.288				0.000

k =	1	2	3	4	5	6	7
f _k =	2000	4000	6000	8000	10000	12000	14000
	f _c	2f _c	3f _c	4f _c	5f _c		

All energy falls into integer multiples of the fundamental, f_c. Therefore no leakage.

MATLAB "-128 TO +128 dBm	2 line spectra Q - N		2 line spectra S - N		2 line spectra Q - S	
	0.000	0.00%	-23.979	#DIV/0!	23.979	100.00%
	-0.001	0.00%	-20.058	#DIV/0!	20.057	100.00%
	-0.008	-0.07%	-10.522	#DIV/0!	10.515	100.00%
	-0.022	-0.36%	-6.099	#DIV/0!	6.078	100.00%
	-0.043	-1.36%	-3.198	#DIV/0!	3.155	100.00%
	-0.071	-7.28%	-1.043	#DIV/0!	0.972	100.00%
	-0.106	13.72%	0.665	#DIV/0!	-0.771	100.00%
	-0.148	6.66%	2.074	#DIV/0!	-2.222	100.00%
		#DIV/0!	3.268	#DIV/0!	0.000	

8	9	10
16000	18000	20000

FOURIER							
SERIES		MATLAB					
1 line spectra		1 line spectra		1 line spectra			
P - O		R - O		P - R			
0.000	0.00%	-0.136	-0.57%	0.136	0.57%	1.000	
-0.001	0.00%	1.978	7.90%	-1.979	-7.90%	0.637	
						0.000	
-0.008	-0.06%	-0.008	-0.06%	0.000	0.00%	0.212	
						0.000	
-0.022	-0.24%	-0.008	-0.08%	-0.014	-0.15%	0.128	
						0.000	
-0.043	-0.69%	-0.008	-0.13%	-0.035	-0.56%	0.091	
						0.000	
-0.071	-1.78%	-0.008	-0.20%	-0.063	-1.55%	0.071	
						0.000	
-0.106	-4.72%	-0.008	-0.35%	-0.098	-4.17%	0.059	
						0.000	
-0.148	-18.76%	-0.008	-0.90%	-0.140	-15.04%	0.050	
						0.000	
	#DIV/0!	-0.008	2.86%	0.265	-100.00%	0.043	
						0.000	
						0.039	
						0.000	
						0.035	
						0.000	
						0.032	
						0.000	
						0.029	
						0.000	
						0.027	
						0.000	
						0.025	
						0.000	
						0.024	
						0.000	
						0.023	
						0.000	