

$$f_c = 2000 \quad X_w(k) = 0.5 * e^{-j\pi k(K-1)/N} \frac{\sin(\pi k K/N)}{\sin(\pi k/N)} - 0.25 * e^{-j\pi(K-1)(k-1)/N}$$

$f_a =$  actual frequency      1st      TERM      2nd

				Sampling Frequency, Fs		
$f_k =$	$k =$	N	K	= 256000		
				Re	Im	MODULUS
0	0	128	64			
2000	1	128	64	0.5	20.36774	20.3739
4000	2	128	64			
6000	3	128	64	0.5	6.778335	6.7968
8000	4	128	64			
10000	5	128	64	0.5	4.053893	4.0846
12000	6	128	64			
14000	7	128	64	0.5	2.881571	2.9246
16000	8	128	64			
18000	9	128	64	0.5	2.226601	2.2821
20000	10	128	64			
22000	11	128	64	0.5	1.806768	1.8747
24000	12	128	64			
26000	13	128	64	0.5	1.513522	1.5940
28000	14	128	64			
30000	15	128	64	0.5	1.296201	1.3893
32000	16	128	64			
34000	17	128	64	0.5	1.127982	1.2338
36000	18	32	16			
38000	19	128	64	0.5	0.993329	1.1121
40000	20	32	16			
42000	21	128	64	0.5	0.882623	1.0144
44000	22	32	16			
46000	23	128	64	0.5	0.789586	0.9346
48000	24	32	16			
50000	25	128	64	0.5	0.709945	0.8683
52000	26	32	16			
54000	27	128	64	0.5	0.640691	0.8127
56000	28	32	16			
58000	29	128	64	0.5	0.579639	0.7655
60000	30	32	16			
62000	31	128	64	0.5	0.525166	0.7251
64000	32	128	64	-9.801E-16	9.8E-16	0.0000
			0			

$$\frac{\sin(\pi(k-1)K/N)}{\sin(\pi(k-1)/N)} - 0.25 * e^{-j\pi(K-1)(k+1)/N} \frac{\sin(\pi(k+1)K/N)}{\sin(\pi(k+1)/N)}$$

TERM
3rd
TERM

1st  
TERM

TOP	BTM	$X_1(f)$	Re	Im	MODULUS
		32.000	-0.2500	10.18387	10.186939
0.5000	0.0245	207.5475			
		0.0000	-0.2500	-10.1839	10.186939
-0.5000	0.0736	23.0979	0.0000	-3.1E-17	6.242E-16
		0.0000	-0.2500	-3.38917	3.3983754
0.5000	0.1224	8.3420	0.0000	-6.1E-17	6.25E-16
		0.0000	-0.2500	-2.02695	2.0423055
-0.5000	0.1710	4.2767	0.0000	-9.2E-17	6.262E-16
		0.0000	-0.2500	-1.44079	1.4623142
0.5000	0.2191	2.6039	0.0000	-1.2E-16	6.28E-16
		0.0000	-0.2500	-1.1133	1.141025
-0.5000	0.2667	1.7572	0.0000	-1.5E-16	6.303E-16
		0.0000	-0.2500	-0.90338	0.937338
0.5000	0.3137	1.2704	0.0000	-1.8E-16	6.331E-16
		0.0000	-0.2500	-0.75676	0.7969861
-0.5000	0.3599	0.9651	0.0000	-2.1E-16	6.364E-16
		0.000	-0.2500	-0.6481	0.694647
0.5000	0.4052	0.7612	0.0000	-2.5E-16	6.403E-16
		0.0000	-0.2500	0.024623	0.2512096
-0.5000	0.4496	0.6184	0.0000	-2.8E-16	6.447E-16
		0.0000	-0.2500	0.075837	0.2612493
0.5000	0.4929	0.5145	0.0000	-3.1E-16	6.497E-16
		0.0000	-0.2500	0.133628	0.283472
-0.5000	0.5350	0.4367	0.0000	-1.2E-15	2.383E-15
		0.0000	-0.2500	0.20517	0.3234109
0.5000	0.5758	0.3770	0.0000	-3.7E-16	6.616E-16
		0.0000	-0.2500	0.304626	0.3940773
-0.5000	0.6152	0.3302	0.0000	4.9E-16	8.226E-16
		0.0000	-0.2500	0.467717	0.5303388
0.5000	0.6532	0.2930	0.0000	-4.3E-16	6.759E-16
		0.0000	-0.2500	0.82414	0.8612235
-0.5000	0.6895	0.2629	0.0000	-1.3E-15	2.007E-15
0.0000	0.7071	0.0000	-0.2500	-0.26258	0.3625603

**2nd**  
**TERM**

TOP	BTM	$X_2(f)$	Re	Im	MODULUS	TOP	BTM
0.2500	-0.0245	25.9434	-0.2500	-10.18387	10.19	-0.2500	0.0245
		16.0000	0.0000	-3.06E-17	0.00	0.0000	0.0491
-0.2500	0.0245	25.9434	-0.2500	-3.389167	3.40	0.2500	0.0736
0.0000	0.0491	0.0000	0.0000	-6.13E-17	0.00	0.0000	0.0980
0.2500	0.0736	2.8872	-0.2500	-2.026946	2.04	-0.2500	0.1224
0.0000	0.0980	0.0000	0.0000	-9.19E-17	0.00	0.0000	0.1467
-0.2500	0.1224	1.0428	-0.2500	-1.440786	1.46	0.2500	0.1710
0.0000	0.1467	0.0000	0.0000	3.67E-17	0.00	0.0000	-0.6521
0.2500	0.1710	0.5346	-0.2500	-1.113301	1.14	-0.2500	0.2191
0.0000	0.1951	0.0000	0.0000	-1.53E-16	0.00	0.0000	0.2430
-0.2500	0.2191	0.3255	-0.2500	-0.903384	0.94	0.2500	0.2667
0.0000	0.2430	0.0000	0.0000	-1.84E-16	0.00	0.0000	0.2903
0.2500	0.2667	0.2197	-0.2500	-0.756761	0.80	-0.2500	0.3137
0.0000	0.2903	0.0000	0.0000	-2.14E-16	0.00	0.0000	0.3369
-0.2500	0.3137	0.1588	-0.2500	-0.648101	0.69	0.2500	0.3599
0.0000	0.3369	0.0000	0.0000	1.86E-16	0.00	0.0000	-0.5033
0.2500	0.3599	0.1206	-0.2500	-0.563991	0.62	-0.2500	0.4052
0.0000	0.3827	0.0000	0.0000	-2.76E-16	0.00	0.0000	0.4276
-0.2500	0.9952	0.0158	-0.2500	0.075837	0.26	0.2500	0.9569
0.0000	0.4276	0.0000	0.0000	-3.06E-16	0.00	0.0000	0.4714
0.2500	0.9569	0.0171	-0.2500	0.133628	0.28	-0.2500	0.8819
0.0000	0.4714	0.0000	0.0000	-1.23E-15	0.00	0.0000	0.5141
-0.2500	0.8819	0.0201	-0.2500	0.20517	0.32	0.2500	0.7730
0.0000	0.5141	0.0000	0.0000	3.53E-16	0.00	0.0000	-0.5782
0.2500	0.7730	0.0261	-0.2500	0.304626	0.39	-0.2500	0.6344
0.0000	0.5556	0.0000	0.0000	4.9E-16	0.00	0.0000	0.5957
-0.2500	0.6344	0.0388	-0.2500	0.467717	0.53	0.2500	0.4714
0.0000	0.5957	0.0000	0.0000	-4.29E-16	0.00	0.0000	0.6344
0.2500	0.4714	0.0703	-0.2500	0.82414	0.86	-0.2500	0.2903
0.0000	0.6344	0.0000	0.0000	-1.35E-15	0.00	0.0000	0.6716
-0.2500	0.2903	0.1854	-0.2500	2.538293	2.55	0.2500	0.0980
0.0000	0.6716	0.0000	0.0000	6.62E-16	0.00	0.0000	-0.5235
0.2500	0.6895	0.0329	-0.2500	-0.23802	0.35	-0.2500	0.7242

## LEAKAGE

				I = INTEGER	N x fc Fs	1
<u>3rd</u> <u>TERM</u>	<u>TOTAL</u>				<u>FOURIER</u> <u>TRANSFORM</u>	
X <sub>3</sub> (f)	Re	Im	MODULUS	X(f)/N	X(f)*T/N	X(f)*T/N dBm
25.9434	-0.5000	32.0000	32.00	0.250	1.250E-04	-48.061
0.0000	0.5000	36.3677	36.37	0.284	1.421E-04	-46.950
2.8872	-0.5000	-13.5730	13.58	0.106		
0.0000	0.5000	6.7783	6.80	0.053	2.655E-05	-61.519
1.0428	-0.5000	-5.4161	5.44	0.042		
0.0000	0.5000	4.0539	4.08	0.032	1.596E-05	-65.942
0.5346	-0.5000	-3.4677	3.50	0.027		
0.0000	0.5000	2.8816	2.92	0.023	1.142E-05	-68.843
0.3255	-0.5000	-2.5541	2.60	0.020		
0.0000	0.5000	2.2266	2.28	0.018	8.914E-06	-70.998
0.2197	-0.5000	-2.0167	2.08	0.016		
0.0000	0.5000	1.8068	1.87	0.015	7.323E-06	-72.706
0.1588	-0.5000	-1.6601	1.73	0.014		
0.0000	0.5000	1.5135	1.59	0.012	6.226E-06	-74.115
0.1206	-0.5000	-1.4049	1.49	0.012		
0.0000	0.5000	1.2962	1.39	0.011	5.427E-06	-75.309
0.0951	-0.5000	-1.2121	1.31	0.010		
0.0000	0.5000	1.1280	1.23	0.010	4.820E-06	-76.340
0.0171	-0.5000	0.1005	0.51	0.016		
0.0000	0.5000	0.9933	1.11	0.009	4.344E-06	-77.242
0.0201	-0.5000	0.2095	0.54	0.017		
0.0000	0.5000	0.8826	1.01	0.008	3.963E-06	-78.041
0.0261	-0.5000	0.3388	0.60	0.019		
0.0000	0.5000	0.7896	0.93	0.007	3.651E-06	-78.752
0.0388	-0.5000	0.5098	0.71	0.022		
0.0000	0.5000	0.7099	0.87	0.007	3.392E-06	-79.391
0.0703	-0.5000	0.7723	0.92	0.029		
0.0000	0.5000	0.6407	0.81	0.006	3.175E-06	-79.966
0.1854	-0.5000	1.2919	1.39	0.043		
0.0000	0.5000	0.5796	0.77	0.006	2.990E-06	-80.486
1.6264	-0.5000	3.3624	3.40	0.106		
0.0000	0.5000	0.5252	0.73	0.006	2.833E-06	-80.957
0.0298	-0.5000	-0.5006	0.71	0.006	2.764E-06	-81.170

$f_c = 2000$                        $T = 0.0005$                        $k = 1 \quad 2$   
 $F_s = 256000$                        $f_k = 2000 \quad 4000$   
 $N = 128$                        $f_c \quad 2f_c$   
 $f_k = k \cdot F_s / N \quad k = 0, 1, 2 \dots$                       All energy falls into integer multiples of  $f_c$

		<u>FOURIER</u>	<u>FOURIER</u>			<u>FOURIER</u>	
1 line	2 line	<u>SERIES</u>	<u>SERIES</u>			<u>SERIES</u>	
spectra	spectra	1 line	2 line	MATLAB	MATLAB	2 line	
$X(f)/N$	$X(f)/N$	spectra	spectra	"0 TO +128	"-128 TO +128	spectra	
dBm	dBm	dBm	dBm	dBm	dBm	<b>AF - AD</b>	
17.960	17.960	23.979	23.9794	23.8430		6.020	25.10%
22.081	19.071	23.067	20.0570	25.0460		0.986	4.92%
13.526	10.515	0.000	0.0000	19.5320			
7.512	4.502	13.525	10.5146	13.5250		6.013	57.18%
5.577	2.566	0.000	0.0000	11.5880			
3.089	0.079	9.088	6.0776	9.1020		5.999	98.70%
1.756	-1.254	0.000	0.0000	7.7680			
0.188	-2.823	6.165	3.1550	6.2000		5.978	189.47%
-0.826	-3.836	0.000	0.0000	5.1860			
-1.967	-4.978	3.982	0.9722	4.0450		5.950	612.03%
-2.782	-5.792	0.000	0.0000	3.2300			
-3.675	-6.686	2.239	-0.7709	2.3370		5.915	-767.31%
-4.354	-7.364	0.000	0.0000	1.6590			
-5.084	-8.095	0.788	-2.2219	0.9280		5.873	-264.31%
-5.663	-8.674			0.349			
-6.278	-9.288			-0.265			#DIV/0!
-6.781	-9.791			-0.768			
-7.309	-10.319			-1.296			
-2.941	-5.952			0.000			
-8.211	-11.222						
-2.411	-5.421						
-9.010	-12.020						
-1.472	-4.483						
-9.722	-12.732						
-0.018	-3.028						
-10.360	-13.370						
2.184	-0.827						
-10.935	-13.946						
5.738	2.728						
-11.455	-14.465						
13.535	10.525						
-11.926	-14.936						
-12.139	-15.149						

3	4	5	6	7	8	9	10
6000	8000	10000	12000	14000	16000	18000	20000
3f <sub>c</sub>	4f <sub>c</sub>	5f <sub>c</sub>					

he fundamental, f<sub>c</sub>. Therefore no leakage.

<u>MATLAB</u>			<u>FOURIER</u>			<u>MATLAB</u>	
2 line spectra			<u>SERIES</u>			1 line spectra	
<b>AH - AD</b>			<b>AE-AC</b>			<b>AG-AC</b>	
-17.960	#DIV/0!	1.000	6.020	25.10%	5.8831	24.67%	
-19.071	#DIV/0!	6.486	0.986	4.27%	2.9648	11.84%	
		0.000					
-4.502	#DIV/0!	0.722	6.013	44.46%	6.0129	44.46%	
		0.000					
-0.079	#DIV/0!	0.261	5.999	66.01%	6.0129	66.06%	
		0.000					
2.823	#DIV/0!	0.134	5.978	96.96%	6.0125	96.98%	
		0.000					
4.978	#DIV/0!	0.081	5.950	149.40%	6.0124	148.64%	
		0.000					
6.686	#DIV/0!	0.055	5.915	264.12%	6.0124	257.27%	
		0.000					
8.095	#DIV/0!	0.040	5.873	744.86%	6.0123	647.88%	
		0.000					
9.288	#DIV/0!	0.030	6.278				
		0.000					
		0.024					
		0.000					
		0.019					
		0.000					
		0.016					
		0.000					
		0.014					
		0.000					
		0.012					
		0.000					
		0.010					
		0.000					
		0.009					
		0.000					
		0.008					
		0.000					