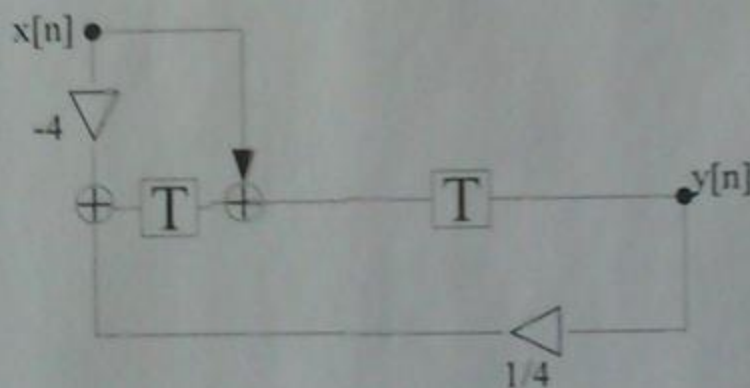


1. System analysis

In the following picture, a linear filter is displayed. All „inner“ quantities following the delay terms were zero at time $t=0$.

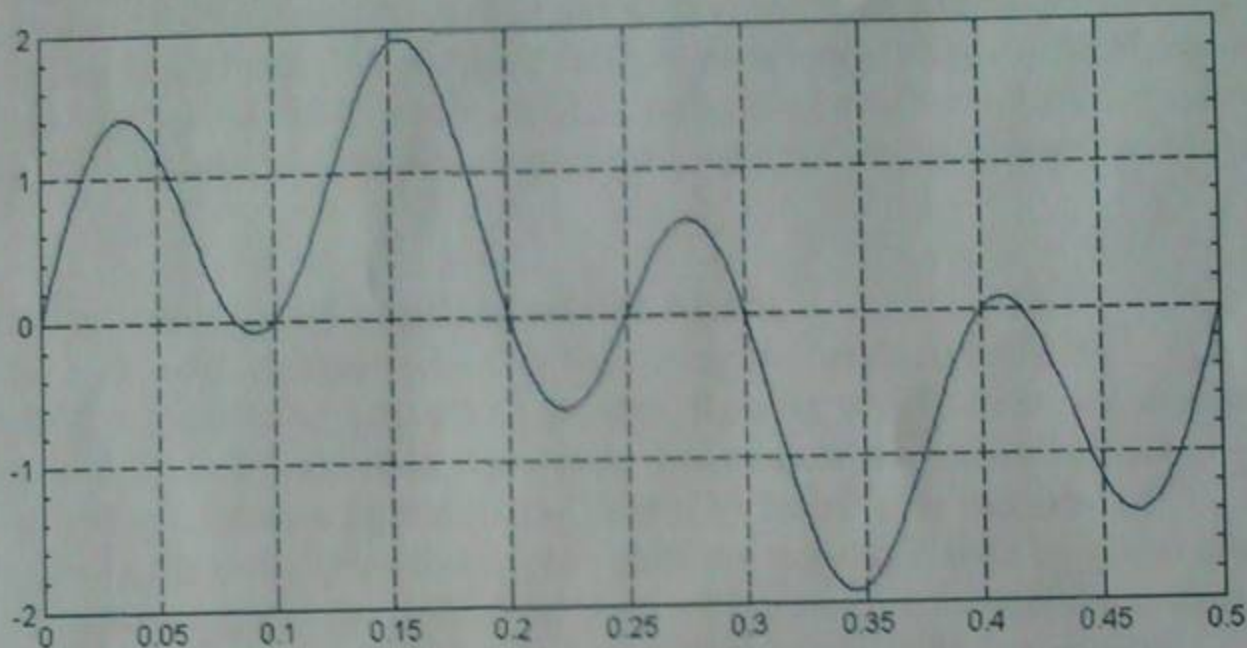


- 1.1. Give the difference equation of the filter.
- 1.2. Give the transfer function $H(z)$ and sketch the pole-zeroes-scheme in the z -plane.
- 1.3. Calculate the impulse response $h[n]$ by inverse Z-Transform from $H(z)$. Take the appropriate area in z -space or $1/z$ space for calculating the inverse Z-Transform efficiently. Solutions which employ m -fold derivatives will not be given credit!

- 1.4. The impulse response $h[n]$ can be calculated in time domain as well. With which series must the system be excited to obtain as $y[n]$ the impulse response?
- 1.5. Excite the system, according to task 1.4. Which recursive equation do you obtain **directly** from the difference equation that you gave in task 1.1.?
- 1.6. Using the recursive equation from task 1.5., calculate the first four values of the impulse response $h[n]$. Calculate also the first four values of $h[n]$ using the formula obtained in task 1.3. Verify that the results are identical. If not, this gives you an indication of possible errors.
- 1.7. The input series $x[n]$ was obtained by sampling an analog signal with the (known) sampling time T . Up to which angular frequency $\omega = \omega(T)$ can the system be used if one wants to reconstruct the analog signal without loss of information (give reasons!)?
- 1.8. Is the system a) stable, b) minimal-phased, c) causal, d) an all-pass? Explain why!

3. Sampling and reconstruction

- 3.1. In the following figure, slightly more than one period of a signal is displayed. Sample with $f_1=10$ Hz and with $f_2=20$ Hz. The first sampling point should be at $t=0$. Print the sampling values in the diagram. Write down in a table the sampled values (read off from the figure) for both sampling frequencies.



- 3.2. The sampling values are weighted with a general (unknown) window $p(t)$. Give the *general* formula for the reconstructed signal $x(t)$, using the reconstruction theorem. The formula for the *true* signal can be guessed / seen by close inspection of the picture. Write it down. If you cannot see it, apply the reconstruction theorem that you just used for this particular example.
- 3.3. With the first sampling at $t=0$, and a window of width $T=0.5$, use the formula that you wrote down in task 3.2 to obtain the reconstructed value $x(t=0.27)$ for **both sampling frequencies**. Compare for both sampling frequencies the values
- after reconstruction with a triangular window, defined as follows: linear increase von $p(0)=0$ to $p(T/2)=2.0$, then linear decrease to $p(T)=0$. [The integral over $p(t)$ is then T , as with rectangular windows]
 - after the picture above, or (better) after the formula for the *true* signal obtained in task 3.2.
- Explain the deviations. Explain what has been „done wrong“.