

of Maxwell's equations are collected in Table 7-2 for easy reference. It is obvious that in non-time-varying cases these equations simplify to the fundamental relations in Table 7-1 for electrostatic and magnetostatic models.

EXAMPLE 7-5 An a-c voltage source of amplitude V_0 and angular frequency ω , $v_c = V_0 \sin \omega t$, is connected across a parallel-plate capacitor C_1 , as shown in Fig. 7-7. (a) Verify that the displacement current in the capacitor is the same as the conduction current in the wires. (b) Determine the magnetic field intensity at a distance r from the wire.

Solution

a) The conduction current in the connecting wire is

$$i_c = C_1 \frac{dv_c}{dt} = C_1 V_0 \omega \cos \omega t \quad (\text{A}).$$

For a parallel-plate capacitor with an area A , plate separation d , and a dielectric medium of permittivity ϵ the capacitance is

$$C_1 = \epsilon \frac{A}{d}.$$

With a voltage v_c appearing between the plates, the uniform electric field intensity E in the dielectric is equal to (neglecting fringing effects) $E = v_c/d$, whence

$$D = \epsilon E = \epsilon \frac{V_0}{d} \sin \omega t.$$

The displacement current is then

$$\begin{aligned} i_D &= \int_A \frac{\partial \mathbf{D}}{\partial t} \cdot d\mathbf{s} = \left(\epsilon \frac{A}{d} \right) V_0 \omega \cos \omega t \\ &= C_1 V_0 \omega \cos \omega t = i_c. \quad \text{Q.E.D.} \end{aligned}$$

b) The magnetic field intensity at a distance r from the conducting wire can be found by applying the generalized Ampère's circuital law, Eq. (7-54b), to contour

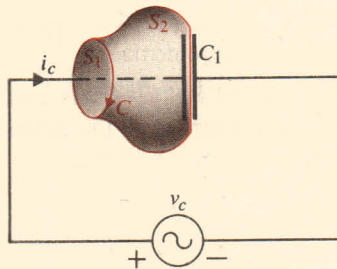


FIGURE 7-7

A parallel-plate capacitor connected to an a-c voltage source (Example 7-5).