Problem 4.33 applies here, because the rotational frequency,

$$\left(3600\,\frac{\text{rev}}{\text{min}}\right)\left(\frac{1}{60}\,\frac{\text{min}}{\text{s}}\right)\left(2\pi\,\frac{\text{rad}}{\text{rev}}\right) = 377 \text{ rad/s}$$

equals the field frequency. Thus, since the effective area is 10 times the loop area,

$$v_{\text{rms}} = \frac{10B_m A\omega_m}{\sqrt{2}} = \frac{10(0.5)(0.20\times 0.16)(377)}{1.414} = 42.66 \text{ V}$$

4.35 A magnetic field, $\mathbf{B} = 0.4\sin(\pi x/2)\cos(\pi y/2)\sin 377t\,\mathbf{a}_z$ (T), passes through the stationary loop $0 < x,\ y < 1$ in the xy plane. Determine the voltage induced in the loop.

$$\psi_m = \left[\int_0^1\int_0^1 0.4\sin\frac{\pi x}{2}\cos\frac{\pi y}{2}\,dx\,dy\right]\sin 377t = \frac{1.6}{\pi^2}\sin 377t \quad \text{(Wb)}$$

$$v = -\frac{d\psi_m}{dt} = -\frac{(1.6)(377)}{\pi^2}\cos 377t = -61.12\cos 377t \quad \text{(V)}$$

4.36 Given a straight conductor Oa (Fig. 4-11) of length ℓ in the xy plane, rotating about O at angular velocity ω_m in a magnetic field $\mathbf{B} = B_0\mathbf{a}_z$, find the induced voltage in the conductor and identify the positive terminal.

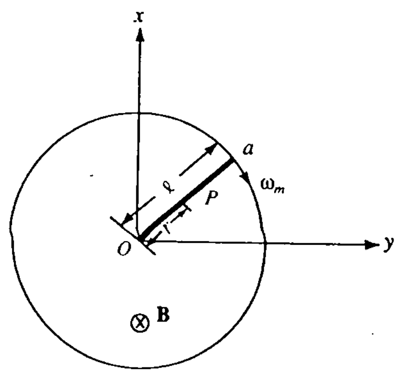

Fig. 4-11

By Problem 4.20, the **E**-field at a point P at a radius r is $\mathbf{E}(r) = \mathbf{u}(r)\times\mathbf{B} = r\omega_m B_0\mathbf{a}_r$, whence

$$v = \int_0^\ell r\omega_m B_0\,dr = \frac{1}{2}\,\omega_m B_0 \ell^2$$

with a as the positive terminal.

4.37 The conductor of Problem 4.36 is fed with a current I at terminal a. Obtain an expression for the torque about O.

The force on an elementary length dr at P (Fig. 4-11) is $d\mathbf{F} = -I\,dr\,\mathbf{a}_r \times B_0\mathbf{a}_z = B_0 I\,dr\,\mathbf{a}_\phi$, and so

$$\mathbf{T} = \int_{r=0}^{\ell} r\mathbf{a}_r \times d\mathbf{F} = \int_0^\ell B_0 Ir\,dr\,(\mathbf{a}_z) = \frac{1}{2}B_0 I\ell^2\mathbf{a}_z$$

The direction $\mathbf{a}_z$ corresponds to clockwise rotation.

4.38 Let the loop of Fig. 4-10 be stationary ($u = 0$) and suppose a current $i = I\sin\omega t$ in the straight conductor. Express the mutual inductance M between the conductor and the loop in terms of the amplitudes of the current and the induced emf around the loop.

The flux through the loop must have the same time dependence as the current: $\psi_m = \Psi_m\sin\omega t$. Now, by definition, $M = \psi_m/i$, or $Mi = \psi_m$, or

$$M\frac{di}{dt} = \frac{d\psi_m}{dt} \qquad \text{or} \qquad M\omega I\cos\omega t = \omega\Psi_m\sin\omega t = V\cos\omega t$$

Thus $M = V/\omega I$.

4.39 Show that Ampère's law for static fields, curl $\mathbf{H} = \mathbf{J}$, is inadequate for time-varying fields.

The divergence of a curl is zero; but charge conservation requires that the divergence of $\mathbf{J}$ equal $-\partial\rho/\partial t \neq 0$.

4.40 Modify curl $\mathbf{H} = \mathbf{J}$ so that it becomes valid for time-varying fields (see Problem 4.39).