C in Fig. 7–7. Two typical open surfaces with rim C may be chosen: (1) a planar disk surface S_1, or (2) a curved surface S_2 passing through the dielectric medium. Symmetry around the wire ensures a constant H_ϕ along the contour C. The line integral on the left side of Eq. (7–54b) is

$$\oint_C \mathbf{H} \cdot d\boldsymbol{\ell} = 2\pi r H_\phi.$$

For the surface S_1, only the first term on the right side of Eq. (7–54b) is nonzero because no charges are deposited along the wire and, consequently, $\mathbf{D} = 0$.

$$\int_{S_1} \mathbf{J} \cdot d\mathbf{s} = i_C = C_1 V_0 \omega \cos \omega t.$$

Since the surface S_2 passes through the dielectric medium, no conduction current flows through S_2. If the second surface integral were not there, the right side of Eq. (7–54b) would be zero. This would result in a contradiction. The inclusion of the displacement-current term by Maxwell eliminates this contradiction. As we have shown in part (a), $i_D = i_C$. Hence we obtain the same result whether surface S_1 or surface S_2 is chosen. Equating the two previous integrals, we find that

$$H_\phi = \frac{C_1 V_0}{2\pi r} \omega \cos \omega t \qquad \text{(A/m)}.$$

7–4 Potential Functions

In Section 6–3 the concept of the vector magnetic potential $\mathbf{A}$ was introduced because of the solenoidal nature of $\mathbf{B}$ ($\nabla \cdot \mathbf{B} = 0$):

$$\boxed{\mathbf{B} = \nabla \times \mathbf{A} \qquad \text{(T).}} \tag{7–55}$$

If Eq. (7–55) is substituted in the differential form of Faraday's law, Eq. (7–1), we get

$$\nabla \times \mathbf{E} = -\frac{\partial}{\partial t}(\nabla \times \mathbf{A})$$

or

$$\nabla \times \left(\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t}\right) = 0. \tag{7–56}$$

Since the sum of the two vector quantities in the parentheses of Eq. (7–56) is curl-free, it can be expressed as the gradient of a scalar. To be consistent with the definition of the scalar electric potential V in Eq. (3–43) for electrostatics, we write

$$\mathbf{E} + \frac{\partial \mathbf{A}}{\partial t} = -\nabla V,$$