Add the vector $\partial\mathbf{D}/\partial t$ (which has the same units, A/m^2, as $\mathbf{J}$, and which vanishes for a static field) to $\mathbf{J}$. Then, because

$$\operatorname{div}\left(\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}\right)=\operatorname{div}\mathbf{J}+\frac{\partial}{\partial t}\operatorname{div}\mathbf{D}=-\frac{\partial\rho}{\partial t}+\frac{\partial\rho}{\partial t}=0$$

the contradiction of Problem 4.39 is avoided. The term $\partial\mathbf{D}/\partial t$ is known as the *displacement current density*, in contrast to $\mathbf{J}$, the *conduction current density*.

4.41 Give in point form *Maxwell's equations* for time-varying electric and magnetic fields.

$$\operatorname{curl}\mathbf{E}=-\frac{\partial\mathbf{B}}{\partial t}\qquad\text{(Faraday's law; fixed surface)}\qquad(1)$$

$$\operatorname{curl}\mathbf{H}=\mathbf{J}+\frac{\partial\mathbf{D}}{\partial t}\qquad\text{(Modified Ampère's law)}\qquad(2)$$

$$\operatorname{div}\mathbf{D}=\rho\qquad\text{(Gauss' law for }\mathbf{D})\qquad(3)$$

$$\operatorname{div}\mathbf{B}=0\qquad\text{(Gauss' law for }\mathbf{B})\qquad(4)$$

4.42 Express Maxwell's equations, (*1*)–(*4*) of Problem 4.41, in integral form.

$$\oint_C \mathbf{E}\cdot d\mathbf{s}=-\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{S}\qquad(1)$$

$$\oint_C \mathbf{H}\cdot d\mathbf{s}=\int_S \mathbf{J}\cdot d\mathbf{S}+\frac{d}{dt}\int_S \mathbf{D}\cdot d\mathbf{S}\qquad(2)$$

$$\int_S \mathbf{D}\cdot d\mathbf{S}=\int_V \rho\,dV\qquad(3)$$

$$\int_S \mathbf{B}\cdot d\mathbf{S}=0\qquad(4)$$

4.43 A portion of a circuit containing a capacitor is shown in Fig. 4-12. Show that the displacement current i_d between the capacitor plates is precisely equal to the conduction current i outside the plates.

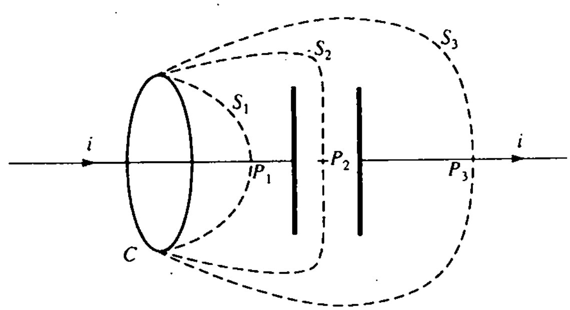

Fig. 4-12

In Fig. 4-12, the three open surfaces S_1, S_2, S_3 all pass through the closed contour C. Noting that $\mathbf{J}=\mathbf{0}$ between the plates, apply (*2*) of Problem 4.42 to C and S_2:

$$\int_C \mathbf{H}\cdot d\mathbf{s}=0+i_d$$

However, for C and S_1 (over which $\partial\mathbf{D}/\partial t=\mathbf{0}$),

$$\int_C \mathbf{H}\cdot d\mathbf{s}=i+0$$

Therefore, $i_d=i$.

4.44 For a material medium characterized by conductivity σ and permittivity ϵ exposed to a sinusoidally varying **E**-field of frequency ω, obtain the ratio of the conduction current density, $|\mathbf{J}_c|$, to the displacement current density, $|\mathbf{J}_d|$.

Let the field (in phasor form) be $\mathbf{E}=E_0e^{j\omega t}$. Then

$$\mathbf{J}_c=\sigma\mathbf{E}=\sigma E_0e^{j\omega t}\qquad\text{and}\qquad -\mathbf{J}_d=\frac{\partial\mathbf{D}}{\partial t}=j\omega\mathbf{D}=j\omega\epsilon E=j\omega\epsilon E_0e^{j\omega t}$$

so that, by division,

$$\frac{|\mathbf{J}_c|}{|\mathbf{J}_d|}=\frac{\sigma}{\omega\epsilon}$$