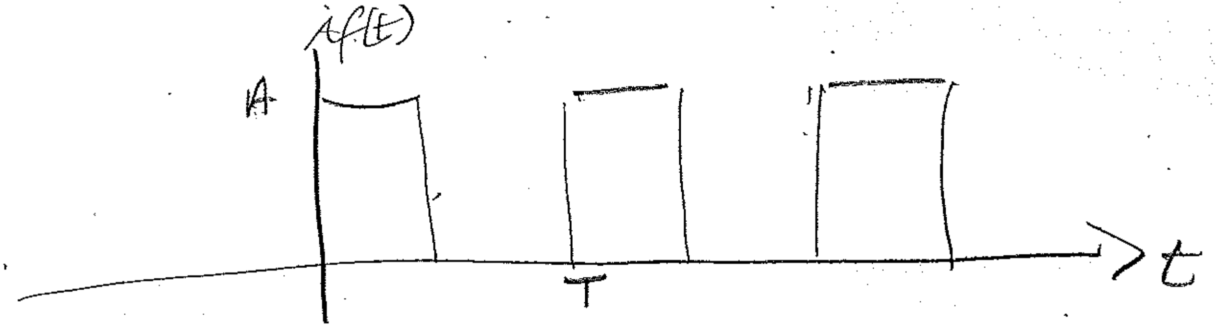


~~f(t) is EVEN i.e. there is symmetry about the y axis.~~

$$f(t) = a_0 + \sum_{n=1}^{\infty}\left[a_n \cos\frac{n\pi t}{T/2} + b_n \sin\frac{n\pi t}{T/2}\right]$$

$$= a_0 + \sum_{n=1}^{\infty}\left[a_n \cos\frac{2\pi n t}{T} + b_n \sin\frac{2\pi n t}{T}\right]$$

a_0 is the average or dc of $f(t)$ and therefore should equal $A/2$

Proof:

$$a_0 = \frac{1}{T}\int_0^T f(t)\,dt$$

$$= \frac{1}{T}\left[\int_0^{T/2} A\,dt + \int_{T/2}^{T} 0\,dt\right]$$

$$= \frac{1}{T}\left[\int_0^{T/2} A\,dt\right] = \frac{1}{T}\Big[At\Big]_0^{T/2}$$

$$= \frac{1}{\cancel{T}}\left[\frac{A\cancel{T}}{2}\right]$$

$$= \frac{A}{2}$$

$$a_n = \frac{1}{T/2} \int_0^T f(t) \cos \frac{n\pi t}{T/2} \, dt$$

$f(t)$ is even, cos is even, product of an even function with an even function is

$$a_n = \frac{2}{T} \left[\int_0^{T/2} f(t) \cos \frac{2\pi n t}{T} \, dt + \overbrace{\int_{T/2}^{T} f(t) \cos \frac{2\pi n t}{T} \, dt}^{=0} \right]$$

$$= \frac{2}{T} \left[\int_0^{T/2} A \cos \frac{2\pi n t}{T} \, dt \right]$$

$$= \frac{2}{T} \left[\frac{AT}{2\pi n} \sin \frac{2\pi n t}{T} \right]_0^{T/2}$$

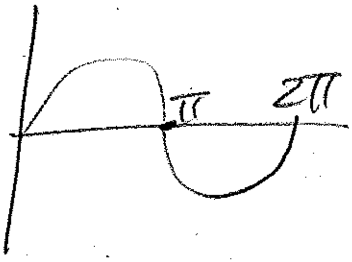
$$= \frac{\cancel{2}}{\cancel{T}} \frac{A\cancel{T}}{\cancel{2}\pi n} \left[\sin \frac{2\pi n t}{T} \right]_0^{T/2}$$

$$= \frac{A}{n\pi} \left[\sin \frac{\cancel{2}\pi n \cancel{T}}{\cancel{2}\cancel{T}} - 0 \right]$$

$$= \frac{A}{n\pi} \sin n\pi \quad = A \frac{\sin n\pi}{n\pi}$$

$$= A \operatorname{sinc}(n\pi)$$

$$a_n = \frac{A}{n\pi} \sin n\pi$$



$n=1$ $\quad a_n = \frac{A}{\pi} \, 0 = 0$

$n=2$ $\quad a_n = \frac{A}{2\pi} \sin 2\pi = 0$

$$a_n = 0$$

$$b_n = \frac{1}{T/2}\int_0^T f(t)\sin\frac{n\pi t}{T/2}\,dt$$

$$= \frac{2}{T}\int_0^T f(t)\sin\frac{2\pi n t}{T}\,dt$$

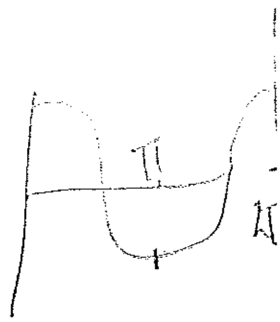
$$= \frac{2}{T}\left[\int_0^{T/2} A\sin\frac{2\pi n t}{T}\,dt + \int_{T/2}^{T} \cancelto{0}{0\cdot\sin\frac{2\pi n t}{T}\,dt}\right]$$

$$= \frac{2}{T}\left[\int_0^{T/2} A\sin\frac{2\pi n t}{T}\,dt\right]$$

$$= \frac{\cancel{2}}{\cancel{T}}\left[\frac{-\cancel{T}A}{\cancel{2}\pi n}\cos\frac{2\pi n t}{T}\right]_0^{T/2}$$

$$= \frac{-A}{n\pi}\left[\cos\frac{\cancel{2}\pi n \cancel{T}}{\cancel{2T}} - 1\right]$$

$$= \frac{A}{n\pi}\left[1 - \cos n\pi\right]$$



$$b_n = \frac{A}{n\pi}\left[1 - \cos n\pi\right]$$

$n=1$ $\quad b_n = \frac{A}{\pi}\left[1 - \cos\pi\right] = \frac{2A}{\pi}$

$n=2$ $\quad b_n = \frac{A}{2\pi}\left[1 - \cos 2\pi\right] = 0$

$n=3$ $\quad b_n = \frac{A}{3\pi}\left[1 - \cos 3\pi\right] = \frac{2A}{3\pi}$

$\vdots$

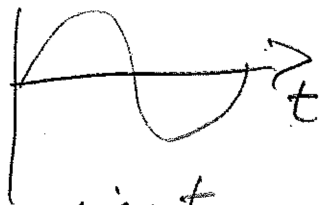
$n=5$ $\quad b_n = \frac{2A}{5\pi}$

$\therefore b_n = \frac{2A}{n\pi}$ for n odd.

$$f(t) = \frac{A}{2} + \sum_{n\ \text{odd}}^{\infty} \frac{2A}{n\pi} \sin \frac{2\pi n t}{T}$$

$$f = \frac{1}{T}$$

$$f(t) = \frac{A}{2} + \sum_{n\ \text{odd}}^{\infty} \frac{2A}{n\pi} \sin 2\pi f n t$$



$\sin \omega t$

$\omega = 2\pi f$

$n=1$ $\quad f(t) = \frac{A}{2} + \frac{2A}{\pi} \sin 2\pi f t$

$n=3$ $\quad f(t) = \frac{A}{2} + \frac{2A}{3\pi} \sin 6\pi f t$

$n=5$ $\quad f(t) = \frac{A}{2} + \frac{2A}{5\pi} \sin 10\pi f t$

From this we can intuitively see the spectrum is simply line spectrum @ odd multiples of f, decreasing in amplitude. The dc component $\frac{A}{2}$ will of course have no effect on the spectrum.