MISCONCEPTIONS (AND CORRECTIONS) IN THE APPLICATIONS OF j-OPERATOR IN ELECTRICAL/ELECTRONICS ENGINEERING

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Abstract: The concepts of j-operator (in electrical/electronics engineering) are recapitulated. Misconceptions on its applications, as presented by certain authors are high-lighted. These misconceptions are corrected and compared with similar (correct) presentations in other texts of electrical and electronics engineering. A call is made for corrections to be effected in texts with these misconceptions, and those similar to them.

Keywords: misconceptions; j-operator; inductance; capacitance; impedance; corrections

1. Introduction

The j-operator, developed from the concepts of complex numbers in mathematics, has wide applications in engineering. In electrical, electronics, communications and computer engineering. It clearly defines electrical quantities in the real and imaginary axes. Representations of these quantities in these axes must be precise and accurate as any error or misconception could lead to technical mis-information.

As a recapitulation, the concepts of j-operator are illustrated in the following sketch:

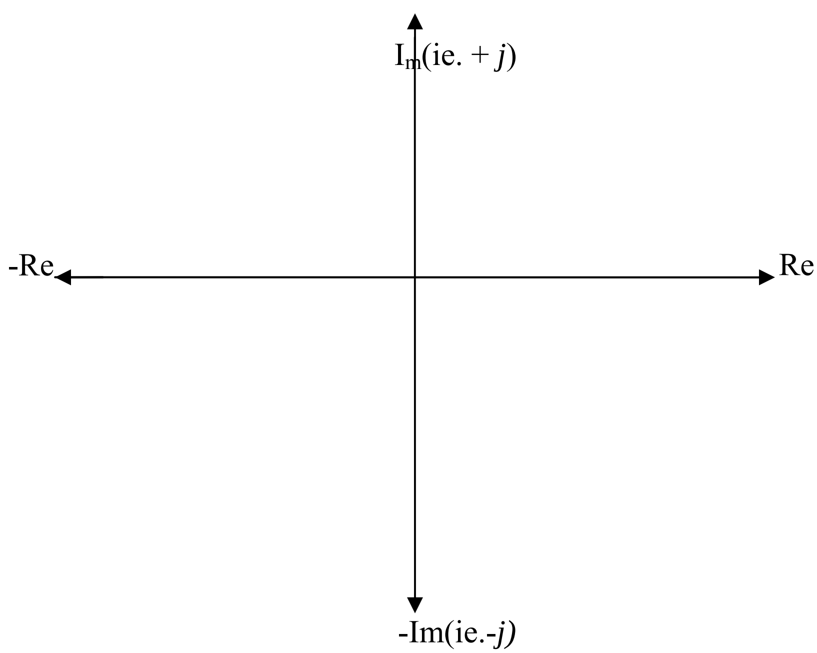


Fig. 1: A sketch of a complex-plane for the *j*-operator.

Where: Re and –Re = Axis for real quantities (real axis)
Im(ie-*j*) and –Im(ie-*j*) = Axis for imaginary quantities (imaginary axis)

2. The *j*-operator theory and its manipulation

The *j*-operator theory is very similar to the complex number theory in mathematics.[1] As already shown in fig.1, in the *j*-operator theory, the absicca (*x*-axis), is known as the real axis while the ordinate (*y*-axis), is known as the imaginary axis, represented by letter "*j*". As already mentioned, this is very similar to the complex number theory in mathematics where we have the following expressions relative to the letter " i " :

$i \times i = i^2 = -1$.--(1)

$i^2 \times i = i^3 = (i^2)(i) = (-1)(i) = -i$---(2)

$i^2 \times i^2 = i^4 = (i^2)(i^2) = (-1)(-1) = 1$--(3)

From eqn. (1), it can be seen that

$i = \sqrt{-1}$ ---(4)

Now, since the *j*-operator concept is similar to the complex numbers concept in mathematics, the following expressions are standard in the *j*-operator theory:[1]

$j \times j = j^2 = -1$--(5)

$j^2 \times j = j^3 = (j^2)(j) = (-1)(j) = -j$---(6)

$j^2 \times j^2 = j^4 = (j^2)(j^2) = (-1)(-1) = 1$--(7)

From eqn.(5), it can be seen that

$j = \sqrt{-1}$ ---(8)

Other manipulations of the *j*-operator are:

$\frac{j}{j} = 1$ --(9)

$\frac{j}{j^2} = \frac{j}{-1} = -j$ ---(10)

$\frac{j}{j^3} = \frac{j}{-j} = -1$ --(11)

$\frac{j}{j^4} = \frac{j}{1} = j$ ---(12)

----------------and so on.

The *j*-operator notation, as illustrated graphically in fig.2 below, with vector $A\angle\theta^o$ (where θ is angular displacement from the positive x-axis, in anticlockwise direction), is capable of covering the entire traditional 360^o circular axial rotation, going in anticlockwise direction.

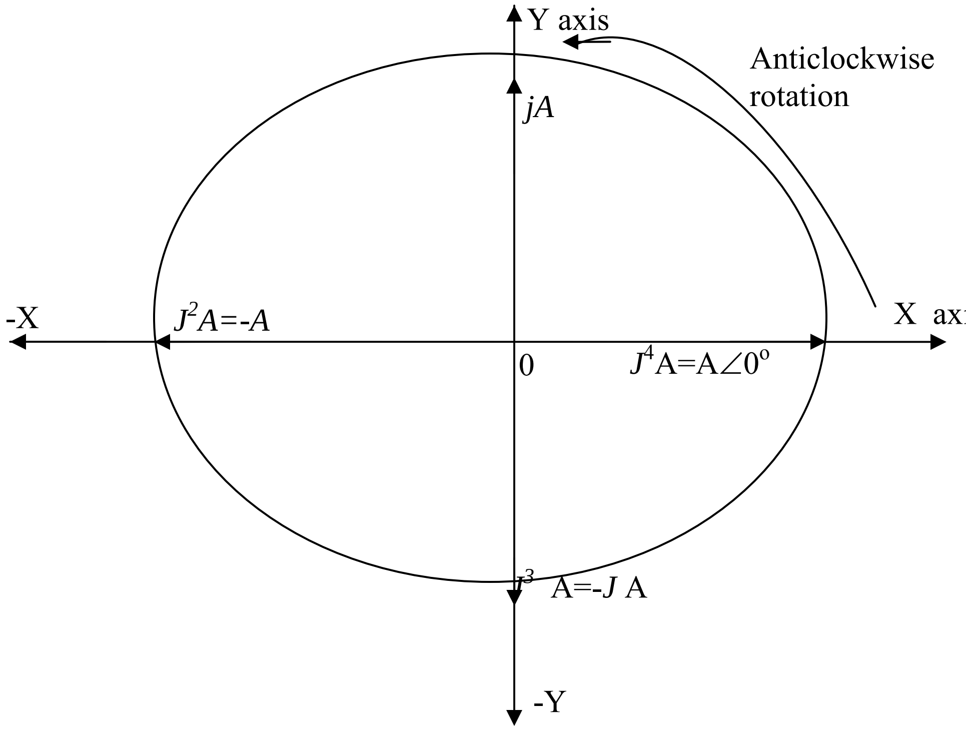


Fig. 2: A graphical illustration of the theory of the *j*-operator

The theory of the *j*-operator above will now be used to study their applications in electrical circuits, as presented by various authors. In the course of the study, the authors of this paper have noted that in electrical engineering, inductive and capacitive quantities are usually expressed with the " *j* " relatively as follows:

inductive quantity (inductive reactance) = $X_L = j\omega L = j2\pi fL$------------------------------(13)

capacitive quantity (capacitive reactance)= $X_c = \frac{1}{j\omega C} = \frac{-j}{2\pi fC}$ ----------------------(14)

3. The *j*-operator, as used by various authors

In the book titled "Electrical Technology, by Edward Hughes, " the author of that book states as follows: "The inductive reactance is expressed by the symbol X_L."[1]

By the authors of this paper, that statement implies that

$X_L = j\text{w}L$---(15)

(Where the symbol "w" denotes "omega" in all cases)

Equation (15) is equally confirmed by R. J. Smith.[2]

Hughes also states as follows: "The capacitive reactance is expressed in ohms and is represented by the symbol X_c." [1] By the authors of this paper, that statement implies that

$X_c = \frac{1}{jwC}$ ---(16)

Equation (16) is equally confirmed by R. J. Smith.[2] Hughes further goes on to state that

$Z_1 = R + jX_L$---(17), [1] and that

$Z_2 = R - jX_C$---(18), [1]

By the authors of this paper, equation (17) and (18) have some misconceptions in them a follows:

(a) The expression in eqn. (17), as it is, should not carry the "*j*" symbol with it because X_L already carries a "*j*" with it as shown in eqn. (15). It should rather be expressed as:

$$Z_1 = R + X_L = R + jwL \text{---} (19)$$

(b) Also, since $X_c = \frac{1}{jwC} = \frac{-j}{wC}$ (as shown in eqn. (15)), then eqn. (18) should be written as:

$$Z_2 = R + X_C = R + \frac{1}{jwC} = R \frac{-j}{wC} \text{---} (20)$$

In a resistance, inductance and capacitance (RLC) circuit in series, eqns.(19) & (20) will then combine to give:

$$Z = R + j(wL - \frac{1}{wC}) \text{---} (21)$$

Equation (21) is what Hughes is trying to state in page 343 of the book when he wrote that

$$Z = R + j(X_L - X_C) \text{---} (22)$$ [1]

4. Other authors with these misconceptions

Edward Hughes is not alone in these misconceptions. For example, in "Higher Electrical Engineering, by Shepherd, et al," the authors state as follows:

"For a pure inductance,

impedance = $jX_L = jwL$"---(23)[3]

They also state as follows in a subsequent paragraph:

" For a pure capacitance,

impedance = $-jX_C = \frac{-j}{wC}$"---(24)[3]

In eqn. (23), there should be no "j" before the $X_{L,,}$ since $X_L = jwL$ already. Similarly, in eqn.(24), there should be no "*-j*" before the X_C since $X_c = \frac{-j}{wC}$ already.

5. Authors presenting the information correctly:

Contrary to Edward Hughes and Shepherd et al, Ralph J. Smith, in his book "Electronics: Circuits and Devices, "states as follows: For a series RLC circuit, the impedance is given by:

$$Z\text{-} = R + jwL + \frac{1}{jwC}$$

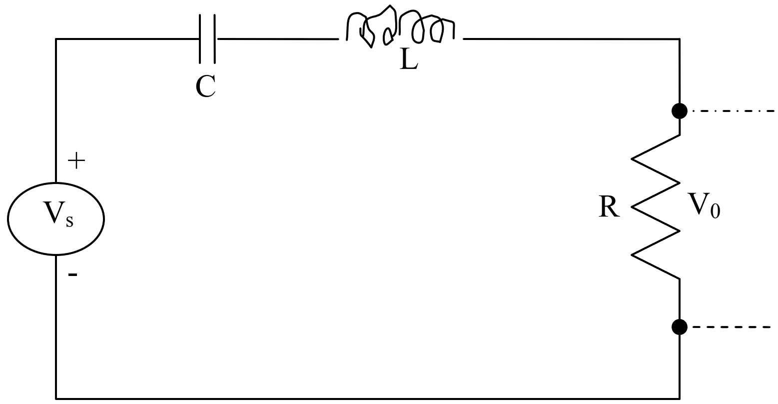
$$Z\text{-} = R + jwL - \frac{j}{wC} = R + j(wL - \frac{1}{wC}) \text{---} (25)$$ [2]

A. H. Morton, in his book "Advanced Electrical Engineering," also states as follows: "The impedance of an RLC series circuit is:

$$Z\text{ -} = R + jwL + \frac{1}{jwC} = R + jwL - \frac{j}{wC}$$

$$= R + j(wL - \frac{1}{wC}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (26)$$ [4]

Similarly, Richard C. Dorf, et al, in their book "Introduction to Electric Circuits, " state thus: " For a series resonance circuit shown below:



The voltage ratio of interest is:

$$\frac{V_0}{V_s} = \frac{R}{R + jwL + \frac{1}{jwC}} = \frac{R}{R + jwL - \frac{j}{wC}}$$

$$\frac{V_0}{V_s} = \frac{R}{R + j(wL - \frac{1}{wC})} \text{--(27)"}$$[5]

It can be seen generally that eqns (25), (26) and the denominator of eqn. (27), all agree with eqn.(21) which the authors of this paper presented; a sharp contrast to eqn. (22) and to the concepts behind eqns. (23) and (24).

6. Discussion

The concepts of the j-operator are very important and powerful that great care must always be taken while applying them to electrical quantities. It should always be noted that a resistive quantity, R, is a real quantity and carries no "*j*" with it. In the contrary, inductive and capacitive quantities carry the "*j*" with them as follows:

Inductive quantity (inductive reactance) = $X_L = jwL$-------------------------------------(28)

Capacitance quantity (capacitance reactance) = $X_c = \frac{1}{jwC} = \frac{-j}{wC}$ -----------------------(29)

These notations should not be mixed up or misconstrued in any algebraic manipulations as shown in the paper.

7. Conclusion

Since the *j*-operator is based on the concepts of complex members in mathematics, its truth and validity is equally based on the truth and the validity of complex numbers and cannot be altered either in error or on purpose. The author of this paper believes that Edward Hughes and J. Shepherd, et al, will see (in good faith), the technical arguments presented in the paper and make necessary corrections (as high-lighted), in subsequent editions of their text books. Similar corrections should equally be effected in other textbooks with similar misconceptions.

References

[1] Hughes, E. "Electrical technology," Longman Group Ltd, London, (1975), pp 294-347

[2] Smith, R. J. “Electronics: Circuits & Devices,” John Wiley & Sons, Inc., New York, (1990), pp. 302-307.

[3] Shepherd, J., et al “higher Electrical Engineering,” Pitman Publishing Ltd., 39 Parker Street, London, WC2B5PB (1978), pp. 35-39.

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[5] Dorf, R. C., et al, “Introduction to Electric Circuits,” John Wiley & Sons, New York, (3rd ed., 1995), pp. 654-655.