Periodic function rule:

$f(t)$ = periodic function

$$L\{f(t)\} = \frac{\int_0^P f(t)\, e^{-st}\, dt}{1 - e^{-Ps}}$$

$\therefore f(t) = \sin at, \quad a > 0.$

If $f(t) = \sin t \qquad P = 2\pi \qquad$ i.e. $a = 1$

$f(t) = \sin 2t \qquad P = \pi \qquad$ i.e. $a = 2$

$\therefore f(t) = \sin at \qquad P = \frac{2\pi}{a}$

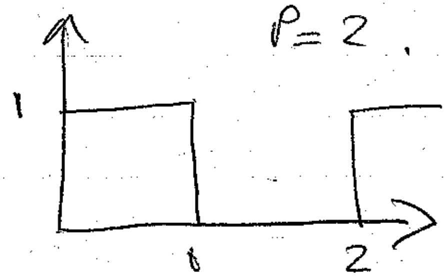
$$L\{f(t)\} = \frac{\int_0^{\frac{2\pi}{a}} \sin at\, e^{-st}\, dt}{1 - e^{-\frac{2\pi}{a}s}}$$

$$\int e^{x}\, dx = e^{x} + c$$

$$\int e^{kx}\, dx = \frac{e^{kx}}{k} + c$$

$f(t) =$ square wave

$P = 2$.



$$\mathcal{L}\{f(t)\} = \int_0^2 f(t)e^{-st}\,dt \Big/ 1-e^{-2s}$$

$$\int_0^2 f(t)e^{-st}\,dt = \int_0^1 f(t)e^{-st}\,dt + \int_1^2 f(t)e^{-st}\,dt$$

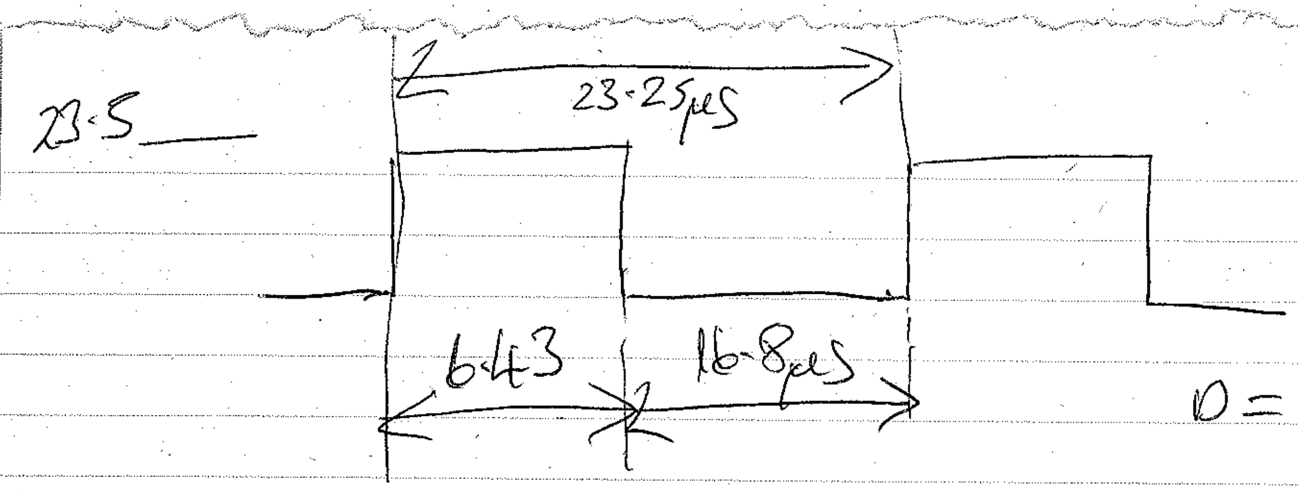
$$= \int_0^1 e^{-st}\,dt + \int_1^2 \cancel{0 \cdot e^{-st}\,dt} = 0$$

$$= \left[\frac{-e^{-st}}{s}\right]_0^1 = \left[\frac{-e^{-s}}{s}\right] - \left[-\frac{1}{s}\right]$$

$$= \frac{1}{s} - \frac{e^{-s}}{s} = \frac{1}{s}(1-e^{-s})$$

$$\mathcal{L}\{f(t)\} = \frac{1}{s}\,\frac{(1-e^{-s})}{(1-e^{-2s})} = \frac{1}{s}\,\frac{\cancel{(1-e^{-s})}}{\cancel{(1-e^{-s})}(1+e^{-s})}$$

$$= \frac{1}{s(1+e^{-s})}.$$



$D = 0.28$

$$f(t) = 23.5 \qquad 0 < t < 6.43$$
$$\quad = 0 \qquad 6.43 < t < 16.8$$

$$L\{f(t)\} = \frac{\int_0^{23.25\mu s} f(t)e^{-st}\,dt}{1 - e^{-23.25 s}}$$

$$\int_0^{23.25} f(t)e^{-st} = \int_0^{6.43} f(t)e^{-st}\,dt \int_{6.43}^{23.25} f(t)e^{-st}\,dt$$

$$\cancel{= \left[\frac{e^{-st}}{23.5}\right]_0^{6.43} + 0}$$

$$\cancel{= \left[\frac{e^{-6.43\times10^{-6}s}}{23.5}\right] - \left[\frac{1}{23.5}\right]}$$

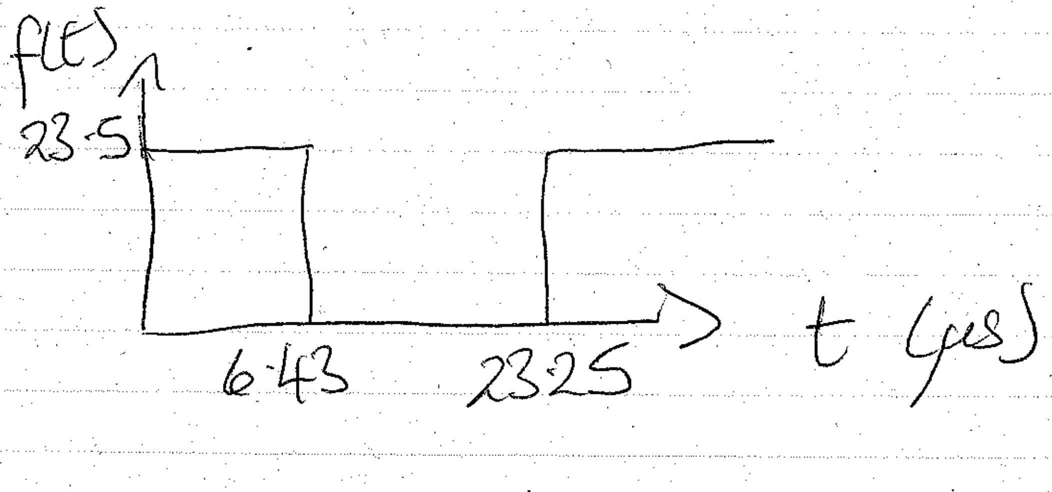
$$\cancel{= \frac{1}{23.5}}$$

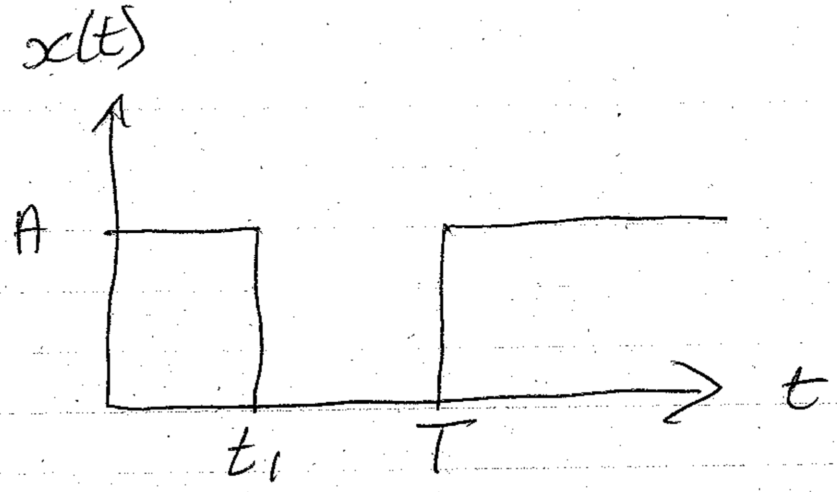
$$= 23.5\left[\frac{-e^{-st}}{s}\right]_0^{6.43\times10^{-6}} + 0$$

$$= 23.5\left[\frac{-e^{-6.43\times10^{-6}s}}{s} - \frac{(-e^0)}{s}\right]$$

$$= \frac{23.5}{s}\left[1 - e^{-6.43\times10^{-6}s}\right]$$

$$\therefore \mathcal{L}\{f(t)\} = \frac{23.5}{s}\ \frac{1-e^{-6.43\times10^{-6}s}}{1-e^{-23.25\times10^{-6}s}}$$



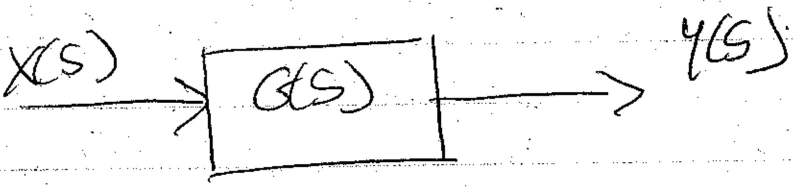


$$L\{x(t)\} = \frac{A}{S}\left\{\frac{1-e^{-t_1 S}}{1-e^{-TS}}\right\}$$

$T = \frac{1}{f}$ $\quad$ f = switching frequency

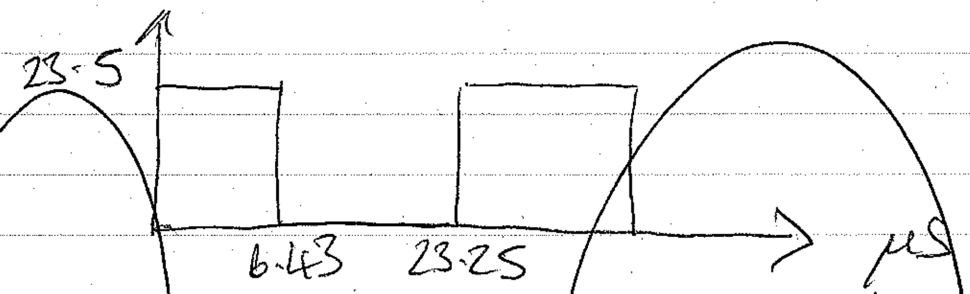
$t_1 = D \cdot T$; D = Duty cycle
D = 0.28

$$L\{x(t)\} = \frac{A}{S}\,\frac{(1-e^{-SD \cdot T})}{(1-e^{-ST})}$$



$$Y(S) = \underbrace{G(S)}_{\text{Transfer Function}} \cdot X(S)$$

$x(t) =$



$$\mathcal{L}\{x(t)\} = \frac{23.5}{s}\,\frac{\left(1-e^{-6.43\times10^{-6}s}\right)}{\left(1-e^{-23.25s}\right)}$$

Transfer function $= \dfrac{1}{s^2LC + \frac{L}{R}s + 1}$

$$Y(s) = \frac{A\left(1-e^{-st_1}\right)}{s\left(1-e^{-sT}\right)}\cdot\frac{1}{(s+a)(s+b)}$$

$$= \frac{A\left(1-e^{-st_1}\right)}{s\left(1-e^{-sT}\right)(s+a)(s+b)}$$

$$\frac{Y(s)}{A} = \frac{1}{s\left(1-e^{-sT}\right)(s+a)(s+b)} - \frac{e^{-st_1}}{s\left(1-e^{-sT}\right)(s+a)(s+b)}$$

$$\frac{Y(s)}{A} = \frac{(1-e^{-st_1})}{s(1-e^{-sT})(s+a)(s+b)}$$

we observe that the inverse laplace transform of this function will be periodic, with period T, because of

$$(1-e^{-sT})$$

Using the Periodic Function Rule

$$f(t) = \text{periodic function}$$

$$L\{f(t)\} = \frac{\int_0^T f(t)e^{-st}\,dt}{1-e^{-sT}}$$

Therefore we find the inverse laplace transform for the first period $f_1(t)$ by ignoring

$(1-e^{-sT})$ term.

$$f_1(t) = L^{-1}\left\{\frac{1-e^{-st_1}}{s(s+a)(s+b)}\right\}$$

$$f_1(t) = L^{-1}\left\{\frac{1}{s(s+a)(s+b)}\right\} - L^{-1}\left\{\frac{e^{-st_1}}{s(s+a)(s+b)}\right\}$$

If $R = 10\,\Omega$

Roots are $a = -2.27\times10^{3} + 21{,}198.59j$

$b = -2.27\times10^{3} - 21{,}198.59j$

$$\frac{1}{s(s+2270-21{,}199j)(s+2270+21{,}199j)}$$

$$= \frac{A_1}{s} + \frac{B}{s+2270-21{,}199j} + \frac{C}{(s+2270+21{,}199j)}$$

$L = 1\times10^{-4}$ $\quad C = 2.2\times10^{-5}$

$$s^2LC + \frac{L}{R}s + 1 = 2.2\times10^{-9}s^2 + 1\times10^{-5}s + 1$$

$$A_1 = \left.\frac{1}{2.2\times10^{-9}s^2 + 1\times10^{-5}s + 1}\right|_{s=0} = 1$$

$A_1 = 1$

$$B = \left.\frac{1}{s(s+2270+21{,}199j)}\right|_{s=-2270+21{,}199j}$$

$$= \frac{1}{(-2270+21{,}199j)(-\cancel{2270}+21{,}199j+\cancel{2270}+21{,}199j)}$$

$$= \frac{1}{(-2270+21{,}199j)(42{,}398j)}$$

$$B = \frac{1}{-96.24\times10^{6}j - 898.79\times10^{6}}$$

$$= \frac{1}{10^{6}(-898.79-96.24j)} \times \frac{(-898.79+96.24j)}{(-898.79+96.24j)}$$

$$= \frac{10^{-6}(-898.79+96.24j)}{\cancel{0.808\times10^{6}} \; 807.823\times10^{3} + 9.262\times10^{3}}$$

$$= \frac{10^{-9}(-898.79+96.24j)}{817.085}$$

$$= 1.224\times10^{-12}(-898.79+96.24j)$$

$$B = -1.0999\times10^{-9} + 1.177\times10^{-10}j$$

$$C = B^{*} = -1.0999\times10^{-9} - 1.177\times10^{-10}j$$

$$L^{-1}\left\{\frac{1}{s(s+a)(s+b)}\right\} = L^{-1}\left\{\frac{1}{s}\right\} + L^{-1}\left\{\frac{-1.0999\times10^{-9}+1.177\times10^{-10}j}{s+2270-21,199j}\right\}$$

$$+L^{-1}\left\{\frac{-1.0999\times10^{-9}-1.177\times10^{-10}j}{s+2270+21,199j}\right\}$$

$$L^{-1}\left\{\frac{1}{s}\right\} = 1 \qquad L^{-1}\left\{\frac{1}{s+a}\right\} = e^{-at}$$

$$L^{-1}\left\{\frac{-1.0999\times10^{-9}+1.177\times10^{-10}j}{s+2270-21,199j}\right\}$$

$$= (-1.0999\times10^{-9}+1.177\times10^{-10}j)\, L^{-1}\left\{\frac{1}{s+2270-21,199j}\right\}$$

$$= (-1.0999\times10^{-9}+1.177\times10^{-10}j)\, e^{-(2270-21,199j)t}$$

$$= (-1.1\times10^{-9}+1.17\times10^{-10}j)\, e^{-2270t+21,199jt}$$

$$= (-1.1\times10^{-9}+1.17\times10^{-10}j)\, e^{-2270t}\cdot e^{21,199jt}$$

Third term ~~Second~~

$$= (-1.1\times10^{-9}-1.17\times10^{-10}j)\, e^{-2270t}\cdot e^{-21,199tj}$$

$$= -1.1\times10^{-9} e^{-2270t} \cdot e^{21{,}199jt} + 1.17\times10^{-10} j\, e^{-2270t} \cdot e^{21{,}199jt}$$

$$+ \qquad\qquad\qquad +$$

$$= -1.1\times10^{-9} e^{-2270t} \cdot e^{-21{,}199jt} - 1.17\times10^{-10} j\, e^{-2270t} \cdot e^{-21{,}199jt}$$

$$= e^{-2270t} \left\{ -1.1\times10^{-9} \left(e^{21{,}199jt} + e^{-21{,}199jt}\right) + 1.17\times10^{-10} j \left(e^{21{,}199jt} - e^{-21{,}199jt}\right) \right.$$

$$= e^{-2270t} \left\{ -2.2\times10^{-9} \frac{\left(e^{21{,}199jt} + e^{-21{,}199jt}\right)}{2} + 2.34\times10^{-10} j \frac{\left(e^{21{,}199jt} - e^{-21{,}199jt}\right)}{2} \right\}$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2} \qquad \sin x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$= e^{-2270t} \left\{ -2.2\times10^{-9} \cos 21{,}199t - 2.34\times10^{-10} \sin 21{,}199t \right\}$$

$$\therefore \mathcal{L}^{-1}\left\{ \frac{1}{s(s+a)(s+b)} \right\} = 1 - e^{-2270t} \left\{ 2.2\times10^{-9} \cos 21{,}199t + 2.34\times10^{-10} \sin 21{,}199t \right\}$$

$$L^{-1}\left\{\frac{e^{-st_1}}{s(s+a)(s+b)}\right\} = ?$$

Find $L^{-1}\left\{e^{-4s}\left(\frac{2}{s^2}-\frac{5}{s}\right)\right\}$

let $F(s) = \frac{2}{s^2} - \frac{5}{s}$

$$L^{-1}\{F(s)\} = L^{-1}\left\{\frac{2}{s^2}-\frac{5}{s}\right\} = 2L^{-1}\left\{\frac{1}{s^2}\right\} - 5L^{-1}\left\{\frac{1}{s}\right\}$$

$$= 2t - 5$$

$$L^{-1}\left\{e^{-4s}\left(\frac{2}{s^2}-\frac{5}{s}\right)\right\} = L^{-1}\left\{e^{-4s}F(s)\right\}$$

Then using the inverse shifting property or time delay property then

$$L^{-1}\left\{e^{-4s}F(s)\right\} = u_4(t)\,f(t-4)$$

$$= u_4(t)\left[2(t-4)-5\right]$$

$$= u_4(t)\left[2t-13\right]$$

$$u_4(t) = 0 \quad 0 < t < 4$$
$$u_4(t) = 1 \quad t > 4$$

$$\therefore u_4(t)\left[2t-13\right] = 0 \text{ if } 0 < t < 4$$
$$= 2t-13 \text{ if } t > 4$$

$$\therefore L^{-1}\left\{\frac{e^{-st_1}}{s(s+a)(s+b)}\right\} = L^{-1}\left\{e^{-st_1} F(s)\right\}$$

where $F(s) = \dfrac{1}{s(s+a)(s+b)}$

$$L^{-1}\left\{e^{-st_1} F(s)\right\} = u_{t_1}(t)\left[f(t_1)\right]$$

$$u_{t_1}(t) = 0 \quad 0 < t < t_1$$
$$u_{t_1}(t) = 1 \quad t > t_1$$

$$f(t) = 1 - e^{-2220t}\left\{2.2\times10^{-9}\cos 21{,}199t + 2.34\times10^{-10}\sin 21{,}199t\right\}$$

$$f(t_1) = 1 - e^{-2220(t-t_1)}\left\{2.2\times10^{-9}\cos 21{,}199(t-t_1) + 2.34\times10^{-10}\sin 21{,}199(t-t_1)\right\}$$

The inverse laplace transform for the first period T $f_1(t)$ where $t_1 = DT$

$$f_1(t) = L^{-1}\left\{\frac{1}{s(s+a)(s+b)}\right\} - L^{-1}\left\{\frac{e^{-st_1}}{s(s+a)(s+b)}\right\}$$

If $R = 10\ \Omega$ then

$$f_1(t) = 1 - e^{-2270t}\left\{2.2\times10^{-9}\cos 21{,}199t + 2.34\times10^{-10}\sin 21199t\right\}$$

$$- \left\{1 - e^{-2270t_1}\left(2.2\times10^{-9}\cos 21{,}199t_1 + 2.34\times10^{-10}\sin 21199t_1\right)\right\}$$

$$= 0 \quad 0 < t < t_1$$

$$= ? \quad t > t_1$$