

What device (bidirectional FET) can I use for the switch?

Initially the switch is off until Vbus is available, the switch then turns on

The switch stays on until the uC switch it off, thus cut off the battery

