

## 7.2 Solenoid Antennas

In the last section we explored ways of making the single-turn loop fill the space available to get the most bandwidth. Now we go in a different direction, by filling the space with many turns. The equivalent circuit for a small  $N$ -turn loop has the same form as that for a single-turn loop, an inductance in series with radiation and loss resistances, shunted by a capacitance, as shown in Figure 2.15. We have analytical help with the radiation resistance and the inductance. In addition to the dimensional parameters in Figure 7.1, let's define the mean diameter as  $D_m = 2b$  and the solenoid length as  $l_c$ . Since the solenoid is electrically small, we can assume that, in receiving mode, the same magnetic field passes through each turn. Then the total open-circuit voltage is just the series addition of that given by Faraday's law for one turn, equation (3.25),  $V_{oc} = -jN\beta A E_{inc}$ . Then the effective height defined in (3.26) is just  $h_e = N\beta A$ . Equation (3.30) relates the effective height to the directivity and radiation resistance. Because the solenoid is small, we may assume its directivity is the same as that for a single turn, 3/2. Solving for the radiation resistance gives us:

$$R_{rad} = 20(NA\beta^2)^2 \quad (7.1)$$

which is  $N^2$  times that for a single turn loop.

Analytical results for the inductance are obtained by assuming the solenoid current can be approximated by a current sheet; that is a way of saying the turns are closely spaced. One simple formula is[1]:

$$L = \frac{N^2 b^2}{9b + 10l_c} \quad \mu H \quad (7.2)$$

However, the dimensions in this formula are inches. I put this in a more general form so that the permeability is explicit and any compatible length unit can be used.

$$L = \frac{3.133\mu_o N^2 b}{0.9 + l_c / b} \quad (7.3)$$

The basic solenoid  $Q$  is

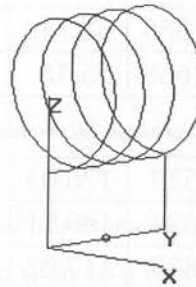
$$Q = \frac{\omega L}{R_{rad}} = \frac{5.984}{(\beta b)^3 (0.9 + l_c / b)} \quad (7.4)$$

Clearly, the number of turns doesn't matter in this expression, but when loss is considered, fewer turns gives less wire length and loss. But, remember the sheet-

current assumption. The  $Q$  goes down with both size parameters. If we make the mean diameter equal to the solenoid length,  $Q = 2.06/(\beta b)^3$ , and for our canonic  $0.1\lambda$  cylinder  $Q = 66.4$ . This is not as good a result as the thin-walled barrel of the last section, and we've probably violated another smallness condition: the total wire length has to be less than  $0.1\lambda$  for the sheet-current assumption [2]. Essentially, the sheet-current theory was developed by people who were interested in inductor design, not antenna design. The inductor is to be used well below its first resonant frequency and to have as high a  $Q$  as possible. The typical inductor has a loss resistance much higher than its radiation resistance. A solenoid used as an antenna should be run at or slightly above its second resonance, where it may have a reasonable radiation resistance and a reactance suitable for the series leg in an impedance-matching circuit.

I've written a wire-list generator in the file `family solenoid.*` to demonstrate the trends in designing solenoid antennas. Figure 7.13 illustrates a four-turn solenoid in free space. As an antenna, the solenoid has to be driven by some wire arrangement and the length of the wire is significant. Notice that the drive wire and the solenoid form an effective one-turn loop perpendicular to the loops forming the solenoid.

Figure 7.13: A four-turn solenoid antenna. It is 0.1 m long by 0.1 m diameter. The axis is up from the  $x$ - $y$  plane by 0.1 m, so that the total feed wire length is 0.2 m.



Since we are looking for electrically small designs, I exercised the programs to find resonances at or below 300 MHz ( $\lambda = 1$  m) for our 0.1-m cylinder. I looked at both free space and perfect ground environments. The results are collected in Table 7.7.