$$\int_0^T x(t)e^{-st}\,dt = A\int_0^{t_1} e^{-st}\,dt + \int_{t_1}^T \cancel{0\,e^{-st}}^{=0}\,dt$$

$$= A\left[\frac{-e^{-st}}{s}\right]_0^{t_1}$$

$$= A\left\{\left[\frac{-e^{-st_1}}{s}\right] - \left[\frac{-1}{s}\right]\right\}$$

$$= A\left[\frac{1}{s}\left(1-e^{-st_1}\right)\right]$$

$$= \frac{A}{s}\left(1-e^{-st_1}\right)$$

$$L\{x(t)\} = \frac{A\left(1-e^{-st_1}\right)}{s\left(1-e^{-sT}\right)} = X(s)$$

$$Y(s) = G(s)\cdot X(s)$$

$$= \frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}\cdot\frac{A}{s}\frac{\left(1-e^{-st_1}\right)}{\left(1-e^{-sT}\right)}$$

$$\frac{Y(s)}{A} = \frac{\omega_n^2 - \omega_n^2 e^{-st_1}}{s\left(s^2+2\zeta\omega_n s+\omega_n^2\right)\left(1-e^{-sT}\right)}.$$