

$$\mathcal{L}^{-1} \left\{ \frac{\omega_n^2 e^{-st_1}}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} \right\} = ?$$

We can use the shifting property theorem:

$$\text{If } \mathcal{L}^{-1}\{G(s)\} = g(t), \text{ then } \mathcal{L}^{-1}\{e^{-st_1} G(s)\} = u(t-t_1) \cdot g(t-t_1)$$

In this case we have:

$$\mathcal{L}^{-1} \left\{ \frac{\omega_n^2 e^{-st_1}}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)} \right\} = \left[ 1 - e^{-(t-t_1)/2RC} \left( \cos \omega(t-t_1) + \frac{\zeta \sin \omega(t-t_1)}{\sqrt{1-\zeta^2}} \right) \right] u(t-t_1)$$

$$\zeta = \frac{1}{2R} \sqrt{\frac{L}{C}}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \sqrt{1 - \frac{L}{4R^2 C}}$$

$$u(t-t_1) = \begin{cases} 0 & t \leq t_1 \\ 1 & t \geq t_1 \end{cases}$$

