

# Log Converters

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Log Converters

One of the most predictable non-linear elements commonly available is the bipolar transistor. The relationship between collector current and emitter base voltage is precisely logarithmic from currents below one picoamp to currents above one milliamp. Using a matched pair of transistors and integrated circuit operational amplifiers, it is relatively easy to construct a linear to logarithmic converter with a dynamic range in excess of five decades.

The circuit in Figure 1 generates a logarithmic output voltage for a linear input current. Transistor Q<sub>1</sub> is used as the non-linear feedback element around an LM108 operational amplifier. Negative feedback is applied to the emitter of Q<sub>1</sub> through divider, R<sub>1</sub> and R<sub>2</sub>, and the emitter base junction of Q<sub>2</sub>. This forces the collector current of Q<sub>1</sub> to be exactly equal to the current through the input resistor. Transistor Q<sub>2</sub> is used as the feedback element of an LM101A operational amplifier. Negative feedback forces the collector current of Q<sub>2</sub> to equal the current through R<sub>3</sub>. For the values shown, this current is 10 μA. Since the collector current of Q<sub>2</sub> remains constant, the emitter base voltage also remains constant. Therefore, only the V<sub>BE</sub> of Q<sub>1</sub> varies with a change of input current. However, the output voltage is a function of the difference in emitter base voltages of Q<sub>1</sub> and Q<sub>2</sub>:

$$E_{OUT} = \frac{R_1 + R_2}{R_2} (V_{BE2} - V_{BE1}) \quad (1)$$

For matched transistors operating at different collector currents, the emitter base differential is given by

$$\Delta V_{BE} = \frac{KT}{q} \log_e \frac{I_{C1}}{I_{C2}} \quad (2)$$

where k is Boltzmann's constant, T is temperature in degrees Kelvin and q is the charge of an electron. Combining these two equations and writing the expression for the output voltage gives

$$E_{OUT} = \frac{-kT}{q} \left[ \frac{R_1 + R_2}{R_2} \right] \log_e \left[ \frac{E_{IN} R_3}{E_{REF} R_{IN}} \right] \quad (3)$$

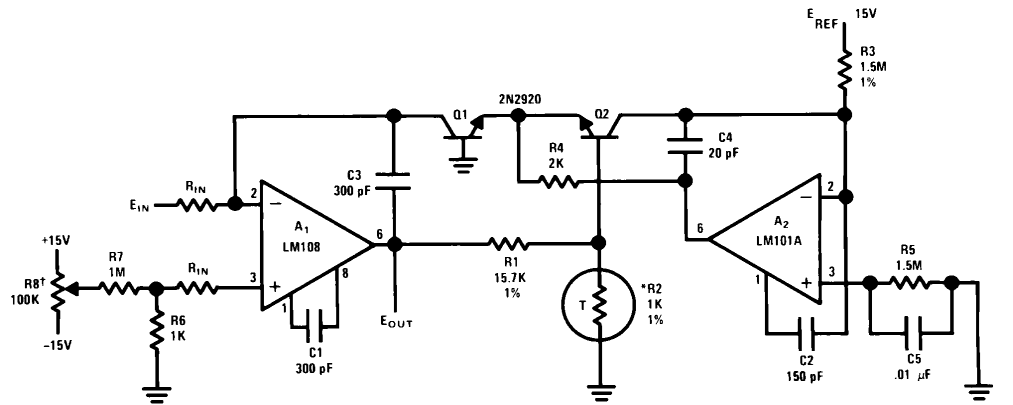
for  $E_{IN} \geq 0$ . This shows that the output is proportional to the logarithm of the input voltage. The coefficient of the log term is directly proportional to absolute temperature. Without compensation, the scale factor will also vary directly with temperature. However, by making R<sub>2</sub> directly proportional to temperature, constant gain is obtained. The temperature compensation is typically 1% over a temperature range of -25°C to 100°C for the resistor specified. For limited temperature range applications, such as 0°C to 50°C, a 430Ω sensistor in series with a 570Ω resistor may be substituted for the 1k resistor, also with 1% accuracy. The divider, R<sub>1</sub> and R<sub>2</sub>, sets the gain while the current through R<sub>3</sub> sets the zero. With the values given, the scale factor is 1V/decade and

$$E_{OUT} = - \left[ \log_{10} \left| \frac{E_{IN}}{R_{IN}} \right| + 5 \right] \quad (4)$$

where the absolute value sign indicates that the dimensions of the quantity inside are to be ignored.

Log generator circuits are not limited to inverting operation. In fact, a feature of this circuit is the ease with which non-inverting operation is obtained. Supplying the input signal to A<sub>2</sub> and the reference current to A<sub>1</sub> results in a log output that is not inverted from the input. To achieve the same 100 dB dynamic range in the non-inverting configuration, an LM108 should be used for A<sub>2</sub>, and an LM101A for A<sub>1</sub>. Since the LM108 cannot use feedforward compensation, it is frequency compensated with the standard 30 pF capacitor.

The only other change is the addition of a clamp diode connected from the emitter of Q<sub>1</sub> to ground. This prevents damage to the logging transistors if the input signal should go negative.



\*1 kΩ (±1%) at 25°C, +3500 ppm/°C.  
Available from Vishay Ultronix, Grand Junction, CO, Q81 Series.  
†Offset Voltage Adjust

FIGURE 1. Log Generator with 100 dB Dynamic Range

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The log output is accurate to 1% for any current between 10 nA and 1 mA. This is equivalent to about 3% referred to the input. At currents over 500  $\mu$ A the transistors used deviate from log characteristics due to resistance in the emitter, while at low currents, the offset current of the LM108 is the major source of error. These errors occur at the ends of the dynamic range, and from 40 nA to 400  $\mu$ A the log converter is 1% accurate referred to the input. Both of the transistors are used in the grounded base connection, rather than the diode connection, to eliminate errors due to base current. Unfortunately, the grounded base connection increases the loop gain. More frequency compensation is necessary to prevent oscillation, and the log converter is necessarily slow. It may take 1 to 5 ms for the output to settle to 1% of its final value. This is especially true at low currents.

The circuit shown in Figure 2 is two orders of magnitude faster than the previous circuit and has a dynamic range of 80 dB. Operation is the same as the circuit in Figure 1, except the configuration optimizes speed rather than dynamic range. Transistor Q<sub>1</sub> is diode connected to allow the use of feedforward compensation<sup>1</sup> on an LM101A operational amplifier. This compensation extends the bandwidth to 10 MHz and increases the slew rate. To prevent errors due to the finite h<sub>FE</sub> of Q<sub>1</sub> and the bias current of the LM101A, an LM102 voltage follower buffers the base current and input current. Although the log circuit will operate without the LM102, accuracy will degrade at low input currents. Amplifier A<sub>2</sub> is also compensated for maximum bandwidth. As with the previous log converter, R<sub>1</sub> and R<sub>2</sub> control the sensitivity; and R<sub>3</sub> controls the zero crossing of the transfer function. With the values shown the scale factor is 1V/decade and

$$E_{OUT} = - \left[ \log_{10} \left| \frac{E_{IN}}{R_{IN}} \right| + 4 \right] \quad (5)$$

from less than 100 nA to 1 mA.

Anti-log or exponential generation is simply a matter of rearranging the circuitry. Figure 3 shows the circuitry of the log converter connected to generate an exponential output from a linear input. Amplifier A<sub>1</sub> in conjunction with transistor Q<sub>1</sub> drives the emitter of Q<sub>2</sub> in proportion to the input voltage. The collector current of Q<sub>2</sub> varies exponentially with the emitter-base voltage. This current is converted to a voltage by amplifier A<sub>2</sub>. With the values given

$$E_{OUT} = 10^{-[E_{IN}]} \quad (6)$$

Many non-linear functions such as X<sup>1/2</sup>, X<sup>2</sup>, X<sup>3</sup>, 1/X, XY, and X/Y are easily generated with the use of logs. Multiplication becomes addition, division becomes subtraction and powers become gain coefficients of log terms. Figure 4 shows a circuit whose output is the cube of the input. Actually, any power function is available from this circuit by changing the values of R<sub>9</sub> and R<sub>10</sub> in accordance with the expression:

$$E_{OUT} = E_{IN}^{\frac{16.7 R_9}{R_9 + R_{10}}} \quad (7)$$

Note that when log and anti-log circuits are used to perform an operation with a linear output, no temperature compensating resistors at all are needed. If the log and anti-log transistors are at the same temperature, gain changes with temperature cancel. It is a good idea to use a heat sink which couples the two transistors to minimize thermal gradients. A 1°C temperature difference between the log and anti-log transistors results in a 0.3% error. Also, in the log converters, a 1°C difference between the log transistors and the compensating resistor results in a 0.3% error.

Either of the circuits in Figures 1 or 2 may be used as dividers or reciprocal generators. Equation 3 shows the outputs of the log generators are actually the ratio of two currents:

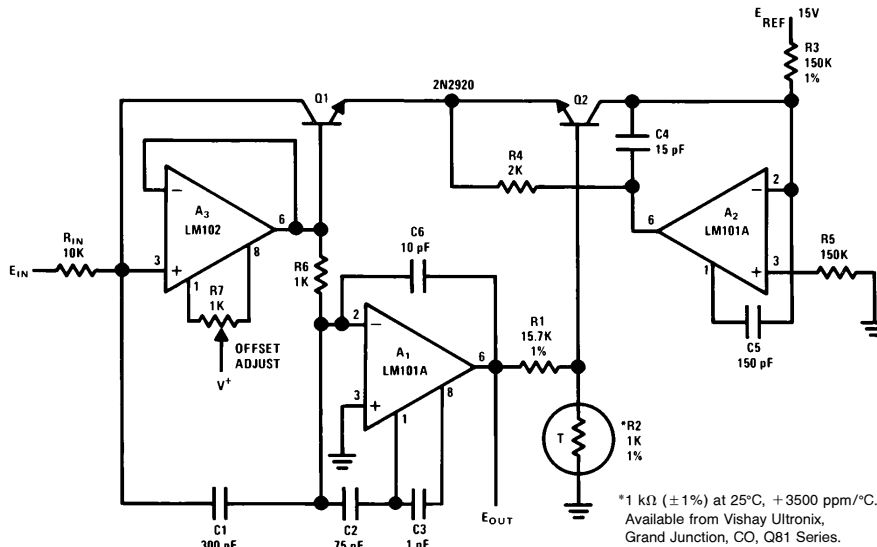


FIGURE 2. Fast Log Generator

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the input current and the current through  $R_3$ . When used as a log generator, the current through  $R_3$  was held constant by connecting  $R_3$  to a fixed voltage. Hence, the output was just the log of the input. If  $R_3$  is driven by an input voltage, rather than the 15V reference, the output of the log generator is the log ratio of the input current to the current through  $R_3$ . The anti-log of this voltage is the quotient. Of course, if the divisor is constant, the output is the reciprocal.

A complete one quadrant multiplier/divider is shown in *Figure 5*. It is basically the log generator shown in *Figure 1* driving the anti-log generator shown in *Figure 3*. The log generator output from  $A_1$  drives the base of  $Q_3$  with a voltage proportional to the log of  $E_1/E_2$ . Transistor  $Q_3$  adds a voltage proportional to the log of  $E_3$  and drives the anti-log transistor,  $Q_4$ . The collector current of  $Q_4$  is converted to an

output voltage by  $A_4$  and  $R_7$ , with the scale factor set by  $R_7$  at  $E_1 E_3/10E_2$ .

Measurement of transistor current gains over a wide range of operating currents is an application particularly suited to log multiplier/dividers. Using the circuit in *Figure 5*, PNP current gains can be measured at currents from  $0.4 \mu\text{A}$  to  $1 \text{ mA}$ . The collector current is the input signal to  $A_1$ , the base current is the input signal to  $A_2$ , and a fixed voltage to  $R_5$  sets the scale factor. Since  $A_2$  holds the base at ground, a single resistor from the emitter to the positive supply is all that is needed to establish the operating current. The output is proportional to collector current divided by base current, or  $h_{FE}$ .

In addition to their application in performing functional operations, log generators can provide a significant increase in

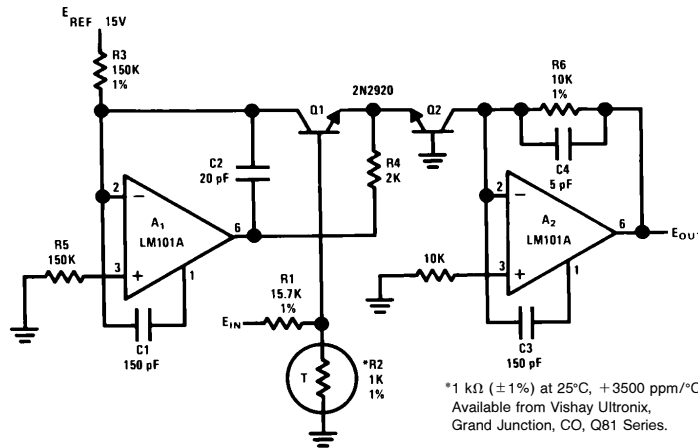


FIGURE 3. Anti-Log Generator

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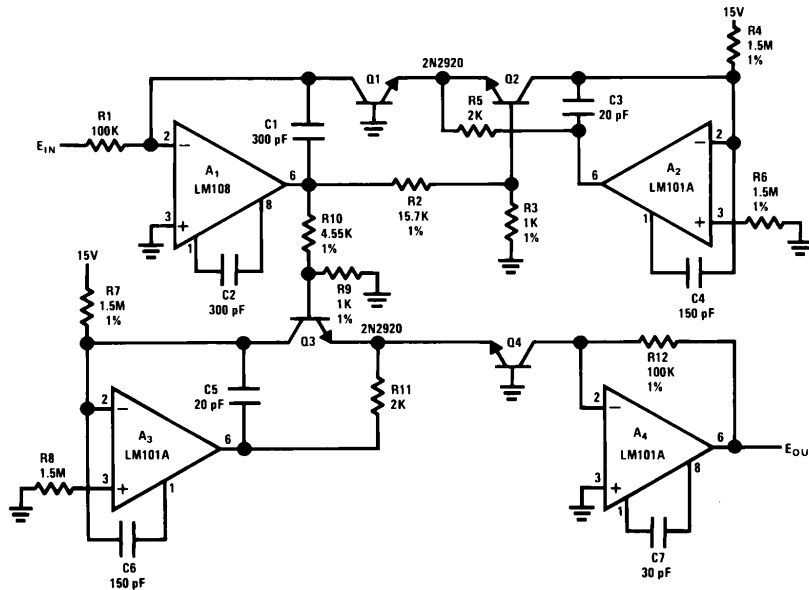


FIGURE 4. Cube Generator

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the dynamic range of signal processing systems. Also, unlike a linear system, there is no loss in accuracy or resolution when the input signal is small compared to full scale. Over most of the dynamic range, the accuracy is a percent-of-signal rather than a percent-of-full-scale. For example, using log generators, a simple meter can display signals with 100 dB dynamic range or an oscilloscope can display a 10 mV and 10V pulse simultaneously. Obviously, without the log generator, the low level signals are completely lost.

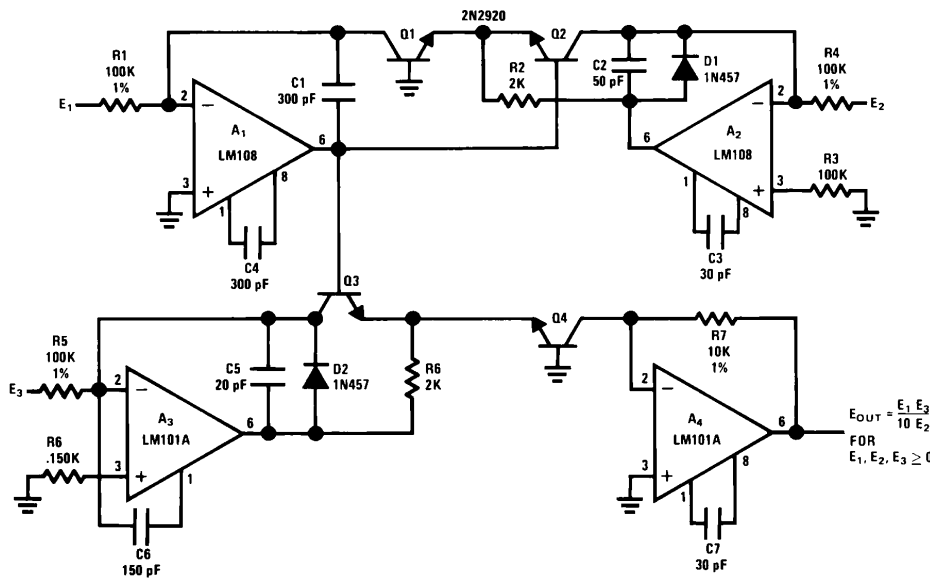
To achieve wide dynamic range with high accuracy, the input operational amplifier necessarily must have low offset voltage, bias current and offset current. The LM108 has a maximum bias current of 3 nA and offset current of 400 pA over a -55°C to 125°C temperature range. By using equal source resistors, only the offset current of the LM108 causes an error. The offset current of the LM108 is as low as many FET amplifiers. Further, it has a low and constant temperature coefficient rather than doubling every 10°C. This results in greater accuracy over temperature than can be achieved with FET amplifiers. The offset voltage may be

zeroed, if necessary, to improve accuracy with low input voltages.

The log converters are low level circuits and some care should be taken during construction. The input leads should be as short as possible and the input circuitry guarded against leakage currents. Solder residues can easily conduct leakage currents, therefore circuit boards should be cleaned before use. High quality glass or mica capacitors should be used on the inputs to minimize leakage currents. Also, when the +15V supply is used as a reference, it must be well regulated.

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2. R. J. Widlar, "Monolithic Operational Amplifiers—The Universal Linear Component", National Semiconductor Corporation, AN-4, April, 1968.



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FIGURE 5. Multiplier/Divider

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